Quiz 6 Key

Math 1220–7

October 26, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. No calculators are allowed on this quiz. Each part of each question is worth 15 points.

1. Find each limit:

   (a) $\lim_{x\to\pi/2} \dfrac{3\sec x + 5}{\tan x}$ (#5 from 8.2)

   This has the indeterminate form $\frac{\infty}{\infty}$, so we can use L'Hôpital's rule:

$$\begin{aligned}\lim_{x\to\pi/2} \frac{3\sec x + 5}{\tan x} &= \lim_{x\to\pi/2} \frac{3\sec x\tan x}{\sec^2 x} \\ &= \lim_{x\to\pi/2} \frac{3\tan x}{\sec x} = \lim_{x\to\pi/2} \frac{3\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \lim_{x\to\pi/2} 3\sin x = 3\end{aligned}$$

   You could have also started by multiplying the numerator and denominator of the original fraction by $\cos x$, leaving you with $\frac{3+5\cos x}{\sin x}$, which you can evaluate without L'Hôpital's rule.

   (b) $\lim_{x\to\infty} x^{1/x}$ (#23 from 8.2)

   This has the indeterminate form ∞^0, so we need to take the natural logarithm before we can apply L'Hôpital's rule:

$$\begin{aligned}\lim_{x\to\infty} \ln x^{1/x} &= \lim_{x\to\infty} \frac{\ln x}{x} \\ &= \lim_{x\to\infty} \frac{1/x}{1} = 0\end{aligned}$$

   To find the limit of $\lim_{x\to\infty} x^{1/x}$, we exponentiate our result to get $e^0 = 1$.

2. Evaluate each improper integral or show that it diverges:

(a) $\displaystyle\int_{-\infty}^{1} \frac{dx}{(2x-3)^3}$ (#15 from 8.3)

$$\begin{aligned}
\int_{-\infty}^{1} \frac{dx}{(2x-3)^3} &= \lim_{a\to-\infty} \int_{a}^{1} (2x-3)^{-3}\,dx \\
&= \lim_{a\to-\infty} -\frac{1}{4}(2x-3)^{-2}\Big|_{a}^{1} \\
&= \lim_{a\to-\infty} -\frac{1}{4}\left((-1)^{-2}-(2a-3)^{-2}\right) \\
&= -\frac{1}{4}
\end{aligned}$$

(b) $\displaystyle\int_{0}^{4} \frac{dx}{(2-3x)^{1/3}}$ (#11 from 8.4)

Since this function is discontinuous at $x = 2/3$, we need to rewrite the function as:

$$\begin{aligned}
\int_{0}^{4} \frac{dx}{(2-3x)^{1/3}} &= \int_{0}^{2/3} (2-3x)^{-1/3}\,dx + \int_{2/3}^{4} (2-3x)^{-1/3}\,dx \\
&= \lim_{t\to 2/3^-} \int_{0}^{t} (2-3x)^{-1/3}\,dx + \lim_{t\to 2/3^+} \int_{t}^{4} (2-3x)^{-1/3}\,dx \\
&= \lim_{t\to 2/3^-} \frac{-1}{2}(2-3x)^{2/3}\Big|_{0}^{t} + \lim_{t\to 2/3^+} \frac{-1}{2}(2-3x)^{2/3}\Big|_{t}^{4} \\
&= -\frac{1}{2}\left(0-2^{2/3}\right) - \frac{1}{2}\left((-10)^{2/3}-0\right) \\
&= \frac{1}{2}\left(2^{2/3}-10^{2/3}\right)
\end{aligned}$$