Quiz 5 Key
Math 1220–7
October 19, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. No calculators are allowed on this quiz. Each part of each question is worth 20 points.

1. Use the method of partial fraction decomposition to calculate:

$$\int \frac{x+8}{(x-1)(x+2)}\,dx$$

$$\frac{x+8}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$
$$x+8 = A(x+2) + B(x-1)$$

Letting $x = -2$, we find that

$$6 = B(-3)$$
$$B = -2$$

Letting $x = 1$, we see that

$$9 = 3A$$
$$A = 3$$

Then,

$$\int \frac{x+8}{(x-1)(x+2)}\,dx = \int \left(\frac{3}{x-1} - \frac{2}{x+2}\right) dx$$
$$= 3\ln\left|x-1\right| - 2\ln\left|x+2\right| + C$$

2. Find each indicated limit:

(a) $\lim_{x \to 0} \frac{x - \sin 2x}{\tan x}$ (#3 from 8.1)

Since this has the indeterminate form 0/0, we can use L'Hôpital's Rule:

$$\begin{aligned} \lim_{x \to 0} \frac{x - \sin 2x}{\tan x} &= \lim_{x \to 0} \frac{1 - 2\cos 2x}{\sec^2 x} \\ &= \frac{1-2}{1} = -1 \end{aligned}$$

(b) $\lim_{x \to 0} \frac{\tan x - x}{\sin 2x - 2x}$ (#15 from 8.1)

Since this also has the indeterminate form 0/0, we can use L'Hôpital's Rule:

$$\begin{aligned} \lim_{x \to 0} \frac{\tan x - x}{\sin 2x - 2x} &= \lim_{x \to 0} \frac{\sec^2 x - 1}{2\cos 2x - 2} \\ &= \lim_{x \to 0} \frac{2\sec^2 x \tan x}{-4\sin 2x} \\ &= \lim_{x \to 0} \frac{2\sec^4 x + 4\sec^2 x \tan^2 x}{-8\cos 2x} \\ &= \frac{2}{-8} = -\frac{1}{4} \end{aligned}$$