Quiz 3 Key
Math 1220–7
September 14, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. Each question is worth 15 points.

Formulas

Euler's Method

$x_n = x_{n-1} + h$

$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$

$$D_x \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}, \quad -1 < x < 1$$

$$D_x \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}, \quad -1 < x < 1$$

$$D_x \tan^{-1} x = \frac{1}{1+x^2}$$

$$D_x \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}, \quad |x| > 1$$

1. Use Euler's Method with $h = 0.25$ to approximate the solution of $y' = xy$ with $y(1) = 3$ over the interval $[1, 2]$.

n	x_n	y_n
0	1	3
1	1.25	$3 + .25 \cdot 1 \cdot 3 = 3.75$
2	1.5	$3.75 + .25 \cdot 1.25 \cdot 3.75 = 4.921875$
3	1.75	$4.921875 + .25 \cdot 1.5 \cdot 4.921875 = 6.767578125$
4	2	$6.767578125 + .25 \cdot 1.75 \cdot 6.767578125 = 9.728393555$

2. Find $\dfrac{dy}{dx}$ if $y = \left(\cos^{-1}(2x^2)\right)\left(\tan^{-1}(e^x)\right)$.

$$\frac{dy}{dx} = \frac{e^x \cos^{-1}(2x^2)}{1+e^{2x}} - \frac{4x \tan^{-1}(e^x)}{\sqrt{1-4x^4}}$$

3. Evaluate $\displaystyle\int \frac{1}{1+4x^2}\,dx$. (#65 from 6.8)

Let $u = 2x$. Then,

$$\int \frac{1}{1+4x^2}\,dx = \frac{1}{2}\int \frac{1}{1+u^2}\,du$$
$$= \frac{1}{2}\tan^{-1} u + C = \frac{1}{2}\tan^{-1}(2x) + C$$

4. Evaluate $\displaystyle\int_0^{\sqrt{2}/2} \frac{1}{\sqrt{1-x^2}}\,dx$. (#61 from 6.8)

$$\int_0^{\sqrt{2}/2} \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x\Big|_0^{\sqrt{2}/2}$$
$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$