**Quiz 2** Key
Math 1220–7
September 7, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. No calculators are allowed on this quiz. Each question is worth 12 points.

1. Evaluate each of the following, given that $D_x\big(a^x\big) = a^x \ln a$:

   (a) $D_x\big(6^{2x}\big)$ (#17 from 6.4)

   $$D_x\big(6^{2x}\big) = 6^{2x}(\ln 6)\cdot 2 = 2(\ln 6)\,6^{2x} \text{ or } 6^{2x}\ln 36$$

   You can also rewrite $6^{2x} = 36^x$ before differentiating.

   (b) $\displaystyle\int x2^{x^2}\,dx$ (#23 from 6.4)

   $$\int x2^{x^2}\,dx = \frac{1}{2}\int 2^u\,du = \frac{1}{2\ln 2}2^u + C = \frac{2^{x^2}}{2\ln 2} + C$$

2. Use logarithmic differentiation to calculate $\dfrac{dy}{dx}$ if $y = \big(x^2+1\big)^{\ln x}$. (#31 from 6.4)

$$\ln y = \ln\left(x^2+1\right)^{\ln x} = \ln x \cdot \ln\left(x^2+1\right)$$

$$\frac{y'}{y} = \frac{1}{x}\ln\left(x^2+1\right) + \frac{2x}{x^2+1}\ln x$$

$$y' = \left(\frac{1}{x}\ln\left(x^2+1\right) + \frac{2x}{x^2+1}\ln x\right)y$$

$$y' = \left(\frac{1}{x}\ln\left(x^2+1\right) + \frac{2x}{x^2+1}\ln x\right)\left(x^2+1\right)^{\ln x}$$

3. Solve the differential equation $\dfrac{dy}{dt} = -6y$, subject to the condition $y(0) = 4$. (#1 from 6.5)

$$\begin{aligned}
\frac{dy}{dt} &= -6y \\
\int \frac{dy}{y} &= -6 \int dt \\
\ln|y| &= -6t + C \quad \text{Exponentiating both sides and using that } y > 0, \\
y &= Ce^{-6t} \quad \text{Using the initial condition,} \\
y &= 4e^{-6t}
\end{aligned}$$

4. Use the fact that $e = \lim_{h \to 0}(1+h)^{1/h}$ to find $\lim_{x \to 0} \left(1+3x\right)^{1/x}$. (#37b from 6.5)

$$\lim_{x \to 0} \left(1+3x\right)^{1/x} = \lim_{x \to 0} \left(\left(1+3x\right)^{1/3x}\right)^{3} = e^{3}$$

5. Solve the following differential equation (Hint: The integrating factor is $e^{\int P(x)\,dx}$.) (#7 from 6.6)

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x}$$

The integrating factor is $e^{\int \frac{1}{x}} = e^{\ln x} = x$, so

$$\begin{aligned}
x\frac{dy}{dx} + y &= 1 \\
D_x(xy) &= 1 \\
\int D_x(xy)\,dx &= \int dx \\
xy &= x + C \\
y &= 1 + \frac{C}{x}
\end{aligned}$$