Section 8.1, Indeterminate Forms of the Type 0/0

Homework: 8.1 #1–23 odds

In this section, we will find limits of the form $\lim_{x\to c} \frac{f(x)}{g(x)}$ where $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0$. In some cases, you have been able to find these. For example,

$$\lim_{x\to 4} \frac{x^2-7x+12}{x^2-16} = \lim_{x\to 4} \frac{(x-4)(x-3)}{(x-4)(x+4)} = \lim_{x\to 4} \frac{x-3}{x+4} = \frac{1}{8}$$

What happens when the numerator and denominator are not polynomials?

To handle this case, we can use **L'Hôpital's Rule**, which says that if $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0$ and if $\lim_{x\to c} \frac{f'(x)}{g'(x)}$ exists (in either the finite or infinite sense), then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

There is a proof of this in the book.

Examples
Find each of the following limits. Make sure that you have an indeterminate form of 0/0 before applying L'Hôpital's Rule.

1. $\lim_{x\to 4} \frac{x^2-7x+12}{x^2-16}$

$$\lim_{x\to 4} \frac{x^2-7x+12}{x^2-16} = \lim_{x\to 4} \frac{2x-7}{2x} = \frac{1}{8}$$

2. $\lim_{x\to 0} \frac{\sin x}{x}$

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1$$

3. $\lim_{x\to 0} \frac{2x+1-\cos x}{\tan x}$

$$\lim_{x\to 0} \frac{2x+1-\cos x}{\tan x} = \lim_{x\to 0} \frac{2+\sin x}{\sec^2 x} = 2$$

4. $\lim_{x\to 0} \frac{\sin x}{x^2-4x}$

$$\lim_{x\to 0} \frac{\sin x}{x^2-4x} = \lim_{x\to 0} \frac{\cos x}{2x-4} = -\frac{1}{4}$$

5. $\lim_{x\to -2} \frac{x^2-x-6}{x^2+4x+4}$

$$\lim_{x\to -2^+} \frac{x^2-x-6}{x^2+4x+4} = \lim_{x\to -2^+} \frac{2x-1}{2x+4} = -\infty$$

6. $\lim_{x\to 0} \dfrac{\sinh x + \tanh x}{e^x - 1}$

$$\lim_{x\to 0} \frac{\sinh x + \tanh x}{e^x - 1} = \lim_{x\to 0} \frac{\cosh x + \operatorname{sech}^2 x}{e^x} = 2$$

7. $\lim_{x\to 0} \dfrac{e^x - \ln(1+x) - 1}{x^2}$

$$\lim_{x\to 0} \frac{e^x - \ln(1+x) - 1}{x^2} = \lim_{x\to 0} \frac{e^x - \frac{1}{1+x}}{2x} = \lim_{x\to 0} \frac{(1+x)e^x - 1}{2x + 2x^2} = \lim_{x\to 0} \frac{e^x + (1+x)e^x}{2 + 4x} = \frac{2}{2} = 1$$