Section 7.6, Strategies for Integration

Homework: 7.6 #1–27 odd, 41, 43

There are many techniques for integration. The options that we have are:

1. u-substitution (Section 7.1)
2. Integration by parts (Section 7.2)
3. Trigonometric substitution (Sections 7.3, 7.4)
4. Partial fraction decomposition (Section 7.5)
5. Integration tables (inside the back cover of the book)
6. Computer or calculator approximations can be helpful, especially for definite integrals.

Examples

1. Find $\displaystyle\int_3^4 \frac{1}{t-\sqrt{2t}}\,dt$

   By the techniques in section 7.4, we should choose $u = \sqrt{2t} = \sqrt{2}t^{1/2}$. Then $du = \frac{\sqrt{2}}{2}t^{-1/2}dt$ and $dt = \sqrt{2}t^{1/2}\,du = u\,du$, and

$$\begin{aligned}\int_3^4 \frac{1}{t-\sqrt{2t}}\,dt &= \int_{\sqrt{6}}^{\sqrt{8}} \frac{u}{\frac{u^2}{2}-u}\,du \\ &= \int_{\sqrt{6}}^{\sqrt{8}} \frac{2}{u-2}\,du \\ &= 2\ln|u-2|\Big|_{\sqrt{6}}^{\sqrt{8}} = 2\ln|\sqrt{8}-2| - 2\ln|\sqrt{6}-2| \approx 1.22283\end{aligned}$$

2. Find $\displaystyle\int \frac{\sqrt{x^2-4x}}{x-2}\,dx$

   First, complete the square in the numerator. This gives:

$$\int \frac{\sqrt{x^2-4x}}{x-2}\,dx = \int \frac{\sqrt{(x-2)^2-4}}{x-2}\,dx$$

   By the techniques in Section 7.4, we should choose $x-2 = 2\sec t$. Then, $dx = 2\sec t\tan t\,dt$, $\sqrt{(x-2)^2-4} = 2\tan t$ and

$$\begin{aligned}\int \frac{\sqrt{(x-2)^2-4}}{x-2}\,dx &= \int \frac{2\tan t}{2\sec t}\cdot 2\sec t\tan t\,dt \\ &= 2\int \tan^2 t\,dt = 2\int \left(\sec^2 t - 1\right)dt \\ &= 2\tan t - 2t + C = 2\tan\sec^{-1}\left(\frac{x-2}{2}\right) - 2\sec^{-1}\left(\frac{x-2}{2}\right) + C \\ &= 2\sqrt{\left(\frac{x-2}{2}\right)^2 - 1} - 2\sec^{-1}\left(\frac{x-2}{2}\right) + C \\ &= \sqrt{x^2-4x} - 2\sec^{-1}\left(\frac{x-2}{2}\right) + C\end{aligned}$$

3. Find $\int \frac{\text{sech}\sqrt{x}}{\sqrt{x}}\,dx$

First, let $u = \sqrt{x}$. Then, $du = \frac{1}{2}x^{-1/2}$, so

$$\begin{aligned}\int \frac{\text{sech}\sqrt{x}}{\sqrt{x}}\,dx &= 2\int \text{sech}u\,du \\ &= 2\int \frac{2}{e^u + e^{-u}}\,du = 4\int \frac{e^u}{e^{2u}+1}\,du\end{aligned}$$

Let $v = e^u$. Then, $dv = e^u\,du$, so

$$4\int \frac{dv}{v^2+1} = 4\tan^{-1} v + C = 4\tan^{-1} e^{\sqrt{x}} + C$$

4. The density of a rod is given as $f(x) = \frac{2}{x^2+1}$. Find c such that the mass from 0 to c is equal to 1.

The mass of the rod from 0 to c is

$$1 = \int_0^c \frac{2}{x^2+1}\,dx = 2\tan^{-1} x\Big|_0^c = 2\tan^{-1} c - 0 \quad \text{Solving for } c\text{, we get}$$

$$1 = 2\tan^{-1} c$$

$$\frac{1}{2} = \tan^{-1} c$$

$$c = \tan\frac{1}{2} \approx 0.5463$$

Note for the homework: For #13–23 odds, the questions come with a part a, then part b. I recommend doing part a first, then using a u-substitution to make part b look like part a. It will make the assignment easier ☺