

Section 7.1, Techniques of Integration

Homework: 7.1 #1–53 odds

This section is a review of integration formulas that we have looked at so far this semester, as well as the ones that you learned in Calculus I. There is a table with formulas on pages 383 and 384 that may be helpful summary.

Examples

Perform each of the following integrations:

1. $\int \frac{3x}{\sin^2(4x^2)} dx$

$$\begin{aligned}\int \frac{3x}{\sin^2(4x^2)} dx &= \int 3x \csc^2(4x^2) dx \quad (\text{Let } u = 4x^2) \\ &= -\frac{3}{8} \cot(4x^2) + C\end{aligned}$$

2. $\int \frac{5e^{3/x^2}}{x^3} dx$

$$\begin{aligned}\int \frac{5e^{3/x^2}}{x^3} dx &= \int 5e^{3x^{-2}} x^{-3} dx \quad (\text{Let } u = 3x^{-2}) \\ &= -\frac{5}{6} e^{3x^{-2}} + C\end{aligned}$$

3. $\int \frac{5}{9 + (2x - 1)^2} dx$

Let $u = (2x - 1)^2$. Then,

$$\int \frac{5}{9 + (2x - 1)^2} dx = \frac{5}{6} \tan^{-1} \left(\frac{2x - 1}{3} \right) + C$$

4. $\int \frac{3x^2 - 4x + 2}{x - 2} dx$

After doing long division, we see that $\frac{3x^2 - 4x + 2}{x - 2} = 3x + 2 + \frac{6}{x - 2}$, so

$$\begin{aligned}\int \frac{3x^2 - 4x + 2}{x - 2} dx &= \int \left(3x + 2 + \frac{6}{x - 2} \right) dx \\ &= \frac{3}{2}x^2 + 2x + 6 \ln |x - 2| + C\end{aligned}$$

5. $\int \frac{2x}{\sqrt{1 - x^4}} dx$

Let $u = x^2$. Then,

$$\int \frac{2x}{\sqrt{1 - x^4}} dx = \sin^{-1}(x^2) + C$$

6. $\int \frac{\sin(\ln 4x^2)}{x} dx$

Let $u = \ln 4x^2$. Then,

$$\int \frac{\sin(\ln 4x^2)}{x} dx = -\frac{1}{2} \cos(\ln 4x^2) + C$$