Section 6.7, Approximations for Differential Equations

Homework: 6.7 #1-7 odds, 11-15 odds

1 Slope Fields

Given a first-order differential equation of the form $y' = f(x, y)$, we can easily find the slope of the line tangent to the curve at any point (x, y) on the coordinate plane. A **slope field** is a graphical representation of the slopes of the tangent lines at many points on the coordinate plane.

Due to the tedious nature of the graphs, you will not be asked to sketch one by hand. Normally, software such as Maple or Mathematica is used.

Examples

1. Given the following slope field, sketch the solution that satisfies $y(0) = 4$. Also find $\lim_{x\to\infty} y(x)$. (Graph given in class)
2. Given the following slope field, sketch the solution that satisfies $y(0) = 4$. Also find the equation of the oblique asymptote. (Graph given in class)

2 Euler's Method

To approximate the solution of the differential equation $y' = f(x, y)$ with initial condition $y(x_0) = y_0$, choose a step size h and repeat the following steps:

1. Set $x_n = x_{n-1} + h$.
2. Set $y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$.

Since this method gives a set of ordered pairs, not a function, it is not very useful when an exact answer is needed. However, it can help to learn about the behavior of the solution, and can be helpful when the differential equation is not solvable.

Example
Use Euler's Method with $h = 0.2$ to approximate the solution to $y' = -2xy$ over $[1, 2]$ if $y(1) = 2$. (#26)

n	x_n	y_n
0	1	2
1	1.2	$2 + .2(-2 \cdot 1 \cdot 2) = 1.2$
2	1.4	$1.2 + .2(-2 \cdot 1.2 \cdot 1.2) = 0.624$
3	1.6	$0.624 + .2(-2 \cdot 1.4 \cdot 0.624) = 0.27456$
4	1.8	$0.27456 + .2(-2 \cdot 1.6 \cdot 0.27456) = 0.0988416$
5	2.0	$0.0988416 + .2(-2 \cdot 1.8 \cdot 0.988416) = 0.027675648$

Note: This equation is separable, so we can solve it as $y = 2e^{1-x^2}$, and in fact $y(2) = 0.099574$. With smaller values of h, this process gives a better estimate.