Section 10.6, Graphs of Polar Equations

Homework: 10.6 #1–37 odds

1 Symmetry in Graphing with Polar Coordinates

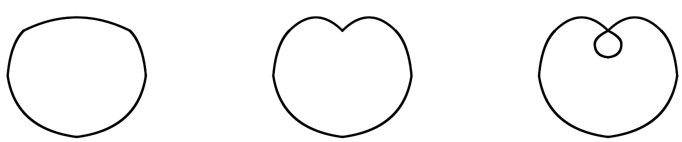
Symmetry is a helpful tool when graphing in Polar Coordinates.

- If replacing (r, θ) by $(r, -\theta)$ gives an equivalent equation, the graph is symmetric with respect to the polar axis (the horizontal axis). For example, if $r = \cos\theta$ and we replace θ by $-\theta$, we get $r = \cos(-\theta) = \cos\theta$ since cosine is an even function. Since this is what we started with, we know that the graph is symmetric with respect to the polar axis.
- If replacing (r, θ) by $(r, \pi - \theta)$ or $(-r, -\theta)$ gives an equivalent equation, the graph is symmetric with respect to the line $\theta = \pi/2$ (the vertical axis). For example, if $r = \sin\theta$, replacing r by $-r$ and θ by $-\theta$ gives $-r = \sin(-\theta) = -\sin\theta$. After we cancel out the negative signs, this is exactly what we started with, so we know that the graph of $r = \sin\theta$ is symmetric with respect to the line $\theta = \pi/2$.
- If replacing (r, θ) by $(r, \pi + \theta)$ or $(-r, \theta)$ gives an equivalent equation, the graph is symmetric with respect to the pole (origin). For example, $r = 5$ and $\theta = \pi/4$ satisfy this criterion.

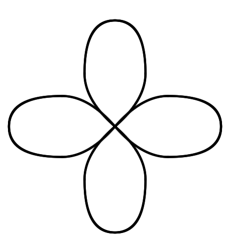
2 Types of Polar Graphs

- The equation for the graph of a **limaçon** has the form $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$. If $a = b$, the graph is called a **cardioid**. The graphs of some limaçons look like:

  $a > b$ $\quad$ $a = b$ $\quad$ $a < b$

  Note: These graphs may be symmetric with respect to the x-axis instead of the y-axis. See Figure 4 on page 543 of the book for examples.
- Figure-eight-shaped curves are called **lemniscates**. The equation has the form $r^2 = \pm a\cos 2\theta$ or $r^2 = \pm a\sin 2\theta$.
- The equation for the graph of a **rose** has the form $r = a\cos n\theta$ or $r = a\sin n\theta$. If n is odd, the graph has n leaves. If n is even, the graph has $2n$ leaves. In general, the graph looks like

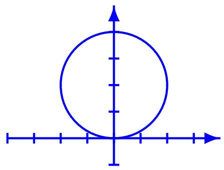
- **Spirals** have equations of the form $r = a\theta$.

Examples

Sketch the graph of each of the following functions. Identify what kind of graph the equation represents, as well as what kind of symmetry exists.

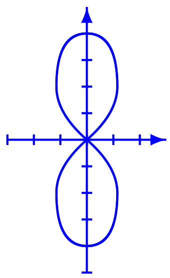
1. $r = 4\sin\theta$

   This is a limaçon that is symmetric about the vertical axis. Specifically, it is a circle of radius 2 centered at the point $(0, 2)$. The graph looks like:

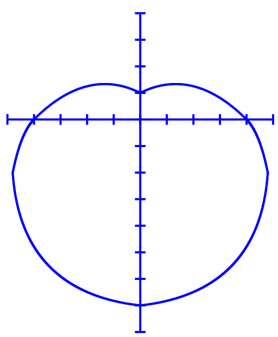
2. $r^2 = -16\cos 2\theta$

   This is a lemniscate that is symmetric with respect to the horizontal axis. The graph looks like:

3. $r = 4 - 3\sin\theta$

   This is the graph of a limaçon that is symmetric with respect to the vertical axis. The graph looks like:

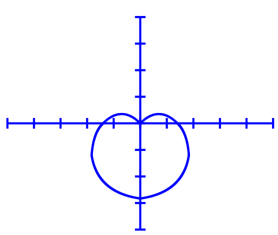
   

4. $r = 2\theta$

   This is the graph of a spiral. There is no symmetry. The graph was sketched in class.

5. $r = \sqrt{2} - \sqrt{2}\sin\theta$

   This is the graph of a cardioid that is symmetric with respect to the vertical axis.

6. $r = 4\cos 2\theta$

   This is the graph of a rose with 4 petals. It is symmetric with respect to the horizontal axis. The graph looks like: