Diagnostic Problems Key

Math 1220-7

Due Friday, August 24, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. No calculators are allowed on this assignment. This assignment is due by the end of class on Friday, August 24.

1. What was the last math class that you took? When did you take it?

2. What other math classes do you need to take?

3. What is your major?

4. Find the derivative of each of the following functions:

   (a) $f(x) = 3x^2 + 2x + \frac{1}{x} + 4\sqrt{x^3}$

   $$f(x) = 3x^2 + 2x + x^{-1} + 4x^{3/2}$$
   $$f'(x) = 6x + 2 - x^{-2} + 6x^{1/2}$$

   (b) $h(t) = 2(4t+5)^5 \sin(3t) + \cos^2 t$

   $$\begin{aligned} h'(t) &= 2 \cdot 5(4t+5)^4 \cdot 4\sin(3t) + 2(4t+5)^5 \cdot 3\cos(3t) - 2\cos t \sin t \\ &= 40(4t+5)^4 \sin(3t) + 6(4t+5)^5 \cos(3t) - 2\cos t \sin t \end{aligned}$$

   (c) $g(x) = \frac{x^3 - 4}{\cos x}$

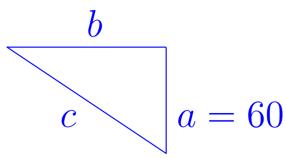
   $$g'(x) = \frac{3x^2 \cos x + (x^3 - 4)\sin x}{\cos^2 x}$$

5. Calculate $D_x y$ by implicit differentiation if x and y satisfy

$$xy + \sin(xy) = 1.$$

$$\begin{aligned}
&xy' + y + \cos(xy)(xy' + y) = 0 \\
&xy' + xy' \cos(xy) = -y - y\cos(xy) \\
&y' = \frac{-y(1+\cos(xy))}{x(1+\cos(xy))} = -\frac{y}{x}
\end{aligned}$$

6. A child is flying a kite. If the kite is 60 feet above the child's hand level and the wind is blowing it away from the child on a horizontal course at 6 feet per second, how fast is the child playing out cord when 100 feet of cord is out? Assume that the cord between the hand and the kite remains straight.



We want to find $\frac{dc}{dt}$ when $c = 100$ ($b = 80$ by the Pythagorean Theorem). We know that $\frac{db}{dt} = 6$ and $\frac{da}{dt} = 0$. By the Pythagorean Theorem,

$$\begin{aligned}
a^2 + b^2 &= c^2 \\
2b\frac{db}{dt} &= 2c\frac{dc}{dt} \\
\frac{dc}{dt} &= \frac{b}{c} \cdot \frac{db}{dt} = \frac{80}{100} \cdot 6 = 4.8\frac{\text{ft}}{\text{sec}}
\end{aligned}$$

7. Let $f(x) = x^3 - 3x$. Find the function's critical points and determine whether they give a local maximum or local minimum by using either the First Derivative Test or the Second Derivative Test. Also give the maximum and minimum values.
$f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$, so there are critical values at $x = \pm 1$. Since $f'(x) > 0$ for $x < -1$ and $x > 1$, and $f'(x) < 0$ for $-1 < x < 1$, there is a local maximum of 2 at $x = -1$ and a local minimum of -2 at $x = 1$.

8. Evaluate the following integrals:

(a) $\displaystyle\int \left(x^3 - 3x + 4 + \frac{1}{x^3} + \sin x + \cos x\right) dx$

$$\int \left(x^3 - 3x + 4 + x^{-3} + \sin x + \cos x\right) dx$$
$$= \frac{x^4}{4} - \frac{3x^2}{2} + 4x - \frac{1}{2x^2} - \cos x + \sin x + C$$

(b) $\displaystyle\int \left(x \sin(x^2)\right) dx$

$$\frac{1}{2}\int \left(2x \sin(x^2)\right) dx = -\frac{\cos(x^2)}{2} + C$$

(c) $\displaystyle\int \left((t^3 + 1)(t^4 + 4t)^5\right) dt$

$$\frac{1}{4}\int \left(4(t^3 + 1)(t^4 + 4t)^5\right) dt = \frac{1}{24}(t^4 + 4t)^6 + C$$

(d) $\displaystyle\int_1^7 \frac{1}{\sqrt{2x + 2}}\, dx$

$$\frac{1}{2}\int_1^7 2(2x + 2)^{-1/2}\, dx = \frac{1}{2} \cdot 2\,(2x + 2)^{1/2}\Big|_1^7 = \sqrt{16} - \sqrt{4} = 2$$

9. Find the function with the derivative $f'(x) = \sin x + 3\cos x + x^2$ such that $f(0) = 5$.

$$\begin{aligned} f(x) &= \int \left(\sin x + 3\cos x + x^2 \right) dx \\ &= -\cos x + 3\sin x + \frac{x^3}{3} + C \\ 5 = f(0) &= -1 + C, \qquad \text{so } C = 6 \\ f(x) &= -\cos x + 3\sin x + \frac{x^3}{3} + 6 \end{aligned}$$

10. Calculate the area between the curve $y = \sin x$ and the x-axis from $x = 0$ to $x = 2\pi$.

$$\begin{aligned} A &= \int_0^{2\pi} \left| \sin x \right| dx \\ &= \int_0^{\pi} \sin x \, dx + \int_{\pi}^{2\pi} (-\sin x) \, dx \\ &= -\cos x \Big|_0^{\pi} + \cos x \Big|_{\pi}^{2\pi} \\ &= 1 - (-1) + 1 - (-1) \\ &= 4 \end{aligned}$$