HW 6 Key

2.7 #9

\[ f(x) = x^2 - 6x + 5. \]

No extremal points, domain is the entire real line.

Crit #‘s are just where \( f'(x) = 0 \).

\[ f'(x) = 2x - 6. \quad f'(x) = 0, \quad 2x - 6 = 0, \quad x = 3. \]

\[ f''(x) = 2, \quad \text{so } f''(3) = 2 \quad \text{and } (3, f(3)) \text{ is a local minimum.} \]

Since we know \( f \) is a parabola, then it's also a global minimum.

2.8 #14

\[ f(x) = \frac{x^2 + 3}{x}. \quad f'(x) = \frac{2x - x - (x^2 + 3)}{x^2} = \frac{2x^2 - x^2 - 3}{x^2} = \frac{x^2 - 3}{x^2}. \]

\[ f'(x) = 0 \text{ when } x = \pm \sqrt{3} \approx \pm 1.7. \]

Also \( f(x) \) is not defined at \( x = 0 \) (vertical asymptote).

\[ f''(x) = \frac{(2x)x^2 - 2x(x^2 + 3)}{x^4} = \frac{2x^3 - 2x^3 + 6x}{x^4} = \frac{6}{x^3}. \]

\[ f''(\sqrt{3}) > 0 \quad \text{MIN} \quad f''(x) = 0 \text{ never. (no inflection points)} \]

\[ f''(\sqrt{3}) < 0 \quad \text{MAX} \]
The resulting graph is the following.

\[ f(\sqrt{3}) = \frac{6}{\sqrt{3}} = 3.4 \quad f(-\sqrt{3}) = -\frac{6}{\sqrt{3}} = -3.4 \]