Worksheet #9

Worksheet #9 is due Thursday, November 12th. You are encouraged to work with other people to solve these problems, but you have to write the solutions down individually. Please turn in these sheets only. You can write at the bottom of each page and/or on the back. Show all the steps and explain your reasoning when necessary: correct solutions with no explanation do not count as complete. Partial credit will be assigned to incomplete answers.

Problem 1. (10 points) Compute the following indefinite integrals using the appropriate techniques

(a) \( \int 2x^3 + \sin x - \frac{1}{x} + 2 \, dx \)
(b) \( \int \frac{x}{\sqrt{x^2 - 2}} \, dx \)
(c) \( \int \frac{t^2}{t^2 + 1} \, dt \)
(d) \( \int x \cos x \, dx \)
Problem 2. (30 points)

(a) Given the following identities
\[ \int \frac{1}{(x+2)(x+5)} \, dx = \frac{1}{3} \ln \left( \frac{x+2}{x+5} \right) + C \]
\[ \int \frac{1}{(x+2)(x-5)} \, dx = -\frac{1}{7} \ln \left( \frac{x+2}{x-5} \right) + C \]

guess the value of the general integral
\[ \int \frac{1}{(x+a)(x+b)} \, dx \quad \text{for} \quad a \neq b \]

and then prove the formula you found using partial fractions. What happens if \( a = b \) instead?

(b) Use integration by parts to evaluate the following integrals
\[ \int \ln x \, dx \quad \int x \ln x \, dx \quad \int x^2 \ln x \, dx; \]

[Hint: think of \( \ln x \) as \( 1 \cdot \ln x \)]. Now guess the value of the general integral
\[ \int x^n \ln x \, dx \quad \text{for} \quad n \geq 0 \]

and then prove the formula you found using differentiation.
Problem 3. (10 points) Let $p \geq 0$ be a real number and consider the improper integral

$$\int_0^1 \frac{1}{x^p} \, dx.$$ 

For what values of $p$ is this integral equal to a finite value? For what values is it infinite? [Hint: Adapt the technique you saw (or will see) in class when computing $\int_1^\infty \frac{1}{x^p} \, dx$ to this case. Is the answer the same?]