Worksheet #4
(50 points)

Worksheet #4 is due Thursday, September 24th. You are encouraged to work with other people to solve these problems, but you have to write the solutions down individually. Please turn in these sheets only. You can write at the bottom of each page and/or on the back. Show all the steps and explain your reasoning when necessary: correct solutions with no explanation do not count as complete. Partial credit will be assigned to incomplete answers.

Problem 1. (10 points)

(a) Compute the following derivatives of polynomials
\[
\frac{d^5}{dx^5}(3x^5+5x^4-2x^2+x-6), \quad \frac{d^7}{dx^7}(-10x^7+3x^6+57x^3+3x^2-2x), \quad \frac{d^8}{dx^8}(x^8+5x^5+2x^3-x+7).
\]

(b) Given
\[
p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0
\]
a generic polynomial of degree \(n\), look at the previous item and come up with a formula for
\[
\frac{d^n}{dx^n} p(x).
\]
Prove your formula as formally as you can.
Problem 2. (20 points) Consider the following picture on a Cartesian plane

(a) Using the informations given by the graph, find the equation for the blue parabola [Hint: remember what kind of polynomial a parabola represents].

(b) Find the equations of the two tangent lines to the parabola through the point $P$ using derivatives. Please do not use the graph as a proof. [Hint: Let $Q$ be any point on the parabola. If its $x$-coordinate is equal to $q$, what is its $y$-coordinate? What is the equation of the tangent line through $Q$?]

(c) Compute the area of the triangle formed by the two tangents and the $x$ axis.
Problem 3. (20 points) Consider an object of mass 1 connected to a spring oscillating without friction on a plane surface. Set up a coordinate system with the origin corresponding to the rest position of the spring. The position of the object at time $t \geq 0$ is given by the following equation

$$x(t) = 5 \cos \left( 3t + \frac{\pi}{4} \right).$$

(a) What is the amplitude of this motion? Does the spring start in resting position at time $t = 0$?

(b) Find the formula for the velocity $v(t)$ and the acceleration $a(t)$. Are there instants when $v(t)$ is equal to 0? If so, what happens to $a(t)$ at those same instants?

(c) Recall that the force exerted by a spring is given by Hook’s law

$$F(t) = -Kx(t).$$

Find the value of $K$ in this case (you can drop the units of measurement here). [Hint: use the laws of dynamics].