Worksheet #1

Worksheet #1 is due Thursday, September 3rd. You are encouraged to work with other people to solve these problems, but you have to write the solutions down individually. Please turn in these sheets only. You can write at the bottom of each page and/or on the back. Show all the steps and explain your reasoning when necessary: correct solutions with no explanation do not count as complete. Partial credit will be assigned to incomplete answers.

Problem 1. (10 points)
(a) Write down the definition of an odd function.

(b) Write down the definition of $f(x) = |x|$ as a piecewise function.

(c) Given the function

$$g(x) = |2\sin(x + \frac{\pi}{4})|,$$

draw its graph in the interval $[-2\pi, 2\pi]$ as precisely as possible. [Hint: draw it in several steps. Start from the graph of $\sin(x)$ and add a transformation at a time]
Problem 2. (20 points) Both Batman and Superman must save the world today. At 6am, Batman jumps on his car and drives away from the Justice League of America Headquarters going due north at 50 mph. An hour later, Superman leaves the Headquarters as well, flying due east at 100 mph.

(a) Ignoring the curvature of the Earth, find a formula for \(d(t)\), the distance (in miles) between Batman and Superman \(t\) hours after 6am. [Hint: The function should be a piecewise function. Why?]

(b) Explain realistically why this function is one-to-one. Then, find the inverse of the function (which is again a piecewise function) and explain its meaning. Be sure to write the function with the correct domain restrictions.

(c) Superman has supersight, so he can see Batman while he’s within 200 miles from him. At what time will Superman lose sight of Batman? Round to the nearest minute.
Problem 3. (20 + 5 points) The surface of a pond has been colonized by a population of bacteria. The initial number of individuals is \( n_0 = 100 \), and the number doubles every three hours.

(a) How many bacteria are there after six hours? And after twelve?

(b) Use an exponential function \( n(t) \) to model this process, where \( t \) is the number of hours. [Hint: pay attention to the fact that the population does not double every hour, but every three]

(c) Find the inverse of the function \( n(t) \) and use it to compute the time at which the population hits 102,400 units.

(d) Bonus question: suppose the density of bacteria on the surface is 1 billion units per square foot. How many square feet does the pond measure approximately, if it takes 3 days to the bacteria to cover it entirely?