Useful Integrals (Take $a \neq 0$ and b to be real, and m and n to be integers.)

Integrals Involving Trigonometric Functions

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C \qquad \int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos^2(ax+b)\,dx = \frac{x}{2} + \frac{1}{4a}\sin\big(2(ax+b)\big) + C$$

$$\int \sin^2(ax+b)\,dx = \frac{x}{2} - \frac{1}{4a}\sin\big(2(ax+b)\big) + C$$

$$\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + C \qquad \int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax + C$$

$$\int x^2\cos ax\,dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax + C$$

$$\int x^2\sin ax\,dx = \frac{2x}{a^2}\sin ax - \frac{a^2x^2-2}{a^3}\cos ax + C$$

$$\int x^m\cos ax\,dx = \frac{x^m\sin ax}{a} - \frac{m}{a}\int x^{m-1}\sin ax\,dx$$

$$\int x^m\sin ax\,dx = -\frac{x^m\cos ax}{a} + \frac{m}{a}\int x^{m-1}\cos ax\,dx$$

$$\int \cos ax\cos bx\,dx = \frac{\sin[(a-b)x]}{2(a-b)} + \frac{\sin[(a+b)x]}{2(a+b)} + C \quad (a^2 \neq b^2)$$

$$\int \sin ax\sin bx\,dx = \frac{\sin[(a-b)x]}{2(a-b)} - \frac{\sin[(a+b)x]}{2(a+b)} + C \quad (a^2 \neq b^2)$$

$$\int \cos ax\sin bx\,dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)} + C \quad (a^2 \neq b^2)$$

Integrals Involving Exponential Functions

$$\int xe^{ax+b}\,dx = \frac{e^{ax+b}}{a^2}(ax-1) + C \qquad \int x^m e^{ax+b}\,dx = \frac{x^m e^{ax+b}}{a} - \frac{m}{a}\int x^{m-1}e^{ax+b}\,dx$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\Big(a\cos bx + b\sin bx\Big) + C \qquad (a^2+b^2 \neq 0)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\Big(a\sin bx - b\cos bx\Big) + C \qquad (a^2+b^2 \neq 0)$$

Identities Involving Bessel Functions (Take $p \geq 0$, $a \neq 0$, $n = 0, 1, \ldots$)

$$\frac{d}{dx}\left[J_0(x)\right] = -J_1(x)$$

$$\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$$

$$\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$$

$$x J_p'(x) + p J_p(x) = x J_{p-1}(x)$$

$$x J_p'(x) - p J_p(x) = -x J_{p+1}(x)$$

$$J_{p-1}(x) - J_{p+1}(x) = 2\, J_p'(x)$$

$$J_{p-1}(x) + J_{p+1}(x) = \frac{2p}{x}\, J_p(x)$$

$$J_n(x) = \frac{1}{\pi} \int_0^{\pi} \cos\left(n\theta - x \sin\theta\right) d\theta$$

$$\int x^{p+1} J_p(x)\, dx = x^{p+1} J_{p+1}(x) + C$$

$$\int x^{-p+1} J_p(x)\, dx = -x^{-p+1} J_{p-1}(x) + C$$

$$\int J_1(x)\, dx = -J_0(x) + C$$

$$\int x J_0(x)\, dx = x J_1(x) + C$$

$$\int J_{p+1}(x)\, dx = \int J_{p-1}(x)\, dx - 2 J_p(x)$$

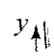
$$x J_{p+1}(x) + p \int J_{p+1}(x)\, dx = \int x J_p(x)\, dx$$

$$\int J_{2n+1}(x)\, dx = -J_0(x) - 2 \sum_{k=1}^{n} J_{2k}(x) + C$$

$$\int_0^a x^{p+1} J_p\left(\frac{\alpha}{a} x\right) dx = \frac{a^{p+2}}{\alpha} J_{p+1}(\alpha)$$

$$\int x J_{2n}(x)\, dx = x J_{2n+1}(x) - 2n J_0(x) - 4n \sum_{k=1}^{n} J_{2k}(x) + C$$

Zeros and Asymptotics of Bessel Functions (Take $n = 0$



$$J_n(x) \sim \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4} - \frac{n\pi}{2}\right) + \mathcal{O}\left(\frac{1}{x^{3/2}}\right)$$

If α_k = kth positive zero of $J_n(x)$, then for large k

$$\alpha_k \approx \frac{\pi}{4} + \frac{(n+1)\pi}{2} + k\pi$$

Improper Integrals (Take $a \neq 0$.)

$$\int_{-\infty}^{\infty} \frac{\sin ax}{x}\, dx = \frac{\pi}{2}, \text{ if } a > 0;\ -\frac{\pi}{2} \text{ if } a < 0.$$

$$\int_0^{\infty} e^{-a^2 x^2} \cos bx\, dx = \frac{\sqrt{\pi}}{2|a|} e^{-b^2/(4a^2)} \quad (b \neq 0)$$

$$\int_{-\infty}^{\infty} \cos(x^2)\, dx = \int_{-\infty}^{\infty} \sin(x^2)\, dx = \sqrt{\frac{\pi}{2}}$$