Mathematics 3150 Practice 2 Summer 2005

1. Non-homogeneous heat problem.
   Find the solution of the heat equation $\dfrac{\partial u}{\partial t} = c^2 \Delta u$ in the unit square ($a = b = 1$) with $c = 1/\pi$. The initial and boundary conditions are given by

   $$f(x, y) = \sin \pi x \sin 2\pi y,\ f_1 = g_1 = g_2 = 0,\ f_2(x) = -\sin \pi x$$

   Hint: you can use this formula

   $$\int_0^1 \sin n\pi y \sinh \pi y dy = \frac{-n(-1)^n}{\pi(1+n^2)} \sinh \pi$$

2. 2-dimensional wave equation.
   Solve the wave equation in the unit square ($a = b = 1$) and with $c = 1/\pi$. The initial conditions are given by

   $$f(x, y) = \sin \pi x \text{ and } g(x, y) = -2 \sin 2\pi x \sin \pi y$$

3. Wave equation in polar coordinates.
   Solve the following wave equations in the unit disk ($a = 1$), with $c = 1$. The initial conditions are given by
   a)

   $$f(r) = -J_0(\alpha_3 r) \text{ and } g(r) = \frac{J_0(\alpha_2 r)}{2}$$

   b)

   $$f(r) = 1 - r^2 \text{ and } g(r) = 3J_0(\alpha_3 r)$$

4. Wave equation in polar coordinates.
   Solve the following wave equations in the unit disk ($a = 1$), with $c = 1$. The initial conditions are given by
   a)

   $$f(r) = (1 - r^2) r^2 \sin 2\theta \text{ and } g(r) = \frac{J_0(\alpha_{02} r)}{2}$$

   b)

   $$f(r) = -3J_0(\alpha_{03} r) + 3J_2(\alpha_{21} r) \sin 2\theta \text{ and } g(r) = -\frac{J_3(\alpha_{32} r)}{4} \cos 3\theta$$