

Mathematics 3150 Practice exam I Summer 2005

1. Sketch the graph and find the Fourier series of the following periodic functions (if the period is not given you have to determine it):
 - (a) The 2π -periodic function defined by: $f(x) = x$ for $-\pi \leq x \leq \pi$.
 - (b) The function $g(x) = |\sin x|$.
 - (c) The 2π -periodic function defined by

$$h(x) = \begin{cases} x + \pi & \text{for } -\pi \leq x \leq -\pi/2 \\ \pi/2 & \text{for } -\pi/2 < x \leq \pi/2 \\ \pi - x & \text{for } \pi/2 < x \leq \pi \end{cases}$$

- (d) $j(x) = \cos^2 x$
- (e) The 2-periodic function defined by $k(x) = |x|$ for $-1 < x < 1$.

2. Find the Fourier sin and cos series of the following functions:

- (a) $f(x) = \sin x$ for $0 < x < \pi$.
- (b) $g(x) = x^2$ for $0 < x < 1$.

3. (a) Prove that for n a positive integer we have

$$\int_0^1 e^x \sin(n\pi x) dx = \frac{n\pi}{1 + n^2\pi^2} (1 - (-1)^n e)$$

(Hint: you have to use 2 integration by parts) (b) Give the solution to the wave equation (with $c = 1/\pi$ and $L = 1$) for the initial conditions $u(x, 0) = e^x$ and $\frac{\partial u}{\partial t}(x, 0) = \sin \pi x - \sin 3\pi x$.

4. Solve the wave equation (with $c = 1$ and $L = \pi$) with homogeneous boundary conditions for the following initial conditions:

- (a) $u(x, 0) = \sin x - 4 \sin 5x$ and $\frac{\partial u}{\partial t}(x, 0) = 0$.
- (b) $u(x, 0) = -3 \cos x$ and $\frac{\partial u}{\partial t}(x, 0) = 2 \sin 3x$.
- (c) $u(x, 0) = \sin x - \sin 3x$ and $\frac{\partial u}{\partial t}(x, 0) = \sin x/2$.
- (d) $u(x, 0) = \cos x$ and $\frac{\partial u}{\partial t}(x, 0) = \sin x$.