Mathematics 1090 EXAM I-Solutions

1. (20 points) Find the slope-intercept equation for the following lines:
(a) The line of equation $2y + 8x - 10 = 0$
Solution: Solve for y:

$$2y + 8x - 10 = 0 \iff 2y = -8x + 10 \iff y = -4x + 5$$

(b) The line passing through the points $(-2, 3)$ and $(3, 1)$
Solution: Slope $m = \dfrac{3-1}{-2-3} = -\dfrac{2}{5}$. Use point slope:

$$y - 3 = -\frac{2}{5}(x - (-2)) = -\frac{2}{5}x - \frac{4}{5} \iff y = -\frac{2}{5}x - \frac{4}{5} + 3 = -\frac{2}{5}x + \frac{11}{5}$$

(c) The line parallel to the line defined by $3y + 12x - 6 = 0$ and passing through $(0, 1)$
Both lines have same slope (because they are parallel) so we compute the slope of the line of equation $3y + 12x - 6 = 0$ by putting the equation in slope-intercept form by solving for y:

$$3y + 12x - 6 = 0 \iff 3y = -12x + 6 \iff y = -4x + 2$$

then this line has slope -4. Now as the line must also pass through $(0, 1)$ which is on the y-axis, we know the y-intercept, so the slope-intercept from of the equation is

$$y = -4x + 1$$

(d) The line perpendicular to $y = 2x - 1$ and passing through $(2, 0)$
Solution: Same thing here: we will compute the slope using the fact that this line is perpendicular to the line $y = 2x-1$ which has slope 2, so the slope of the perpendicular is $m = -1/2$. Now we use point slope:

$$y - 0 = -\frac{1}{2}(x - 2) \iff y = -\frac{1}{2}x + 1$$

2. (25 points) Each ounce of substance A supplies 10% of the nutrition a patient needs and substance B supplies 20%. A patient must be fed by both substances, and moreover you must respect the digestive rule: the given quantity of substance B must be 3/4 of the quantity of substance A. How many ounces of both substances must you provide to this patient to provide him 100% of the required nutrition?
Solution: x is the quantity of A and y the quantity of B. We must set up the equations from the requirements: first we must give 100% of the nutrition, which means that we must satisfy

$$10\%x + 20\%y = 100\% \iff 0.1x + 0.2y = 1 \iff x + 2y = 10$$

I multiplied the last equation by 10 to avoid decimals.
Now the digestive rule reads

$$y = \frac{3}{4}x$$

So we obtained the following system

$$\begin{cases} x + 2y = 10 \\ y = \frac{3}{4}x \end{cases}$$

We solve it by substitution, more precisely we use the second equation to substitute $(3/4)x$ for y in the first one, and then we get

$$x + 2\frac{3}{4}x = 10 \iff x + \frac{3}{2}x = 10 \iff \frac{5}{2}x = 10 \iff x = \frac{2}{5}10 = 4$$

and then $y = (3/4) \times 4 = 3$. So we must give 4 oz of A and 3 of B.

3. (25 points) A company selling washers has fixed costs of \$225 and variable costs of 200+0.2x per unit. The selling price for a washer is \$214.
a) Write the formula for the cost function $C(x)$.
Solution: We get

$$C(x) = 225 + x(200 + 0.2x) = 0.2x^2 + 200x + 225$$

b) Write the profit function and find the break-even points.
Solution: The revenue function is $R(x) = 214x$ so the profit function is

$$P(x) = R(x) - C(x) = 214x - (0.2x^2 + 200x + 225) = -0.2x^2 + 14x - 225$$

The break-even points are given by the equation $P(x) = 0$ so let's solve it:

$$P(x) = 0 \iff -0.2x^2 + 14x - 225 = 0$$

We compute the discriminant: $\Delta = 14^2 - 4(-225)(-0.2) = 196 - 180 = 16$ so we have 2 solutions given by the formulae:

$$x_1 = \frac{-14 + \sqrt{16}}{2(-0.2)} = \frac{-14 + 4}{-0.4} = 25 \text{ and } x_2 = \frac{-14 - \sqrt{16}}{2(-0.2)} = 45$$

So the 2 break-even point occur when producing either 25 or 45 units.

4. (15 points) The following laws for demand and supply are given for a certain type of production:

$$\begin{cases} 2p - q = 19 \\ p + q = 35 \end{cases}$$

Moreover there is a taxation of \$3 on the suppliers. Find the market equilibrium with tax.
Solution: First find the law of supply and put it in the form $p = \dots$. Let's solve for p in the first equation:

$$2p - q = 19 \iff 2p = q + 19 \iff p = q/2 + 9.5$$

this is a law of supply as the slope is positive (slope is 1/2). So we apply the tax and get the new law of supply

$$p = q/2 + 9.5 + 3 = q/2 + 12.5$$

So the new system of laws is

$$\begin{cases} p = q/2 + 12.5 \\ p + q = 35 \end{cases} \iff \begin{cases} p = q/2 + 12.5 \\ p = -q + 35 \end{cases}$$

We solve it by substitution to find the market equilibrium:

$$q/2 + 12.5 = -q + 35 \iff q + 25 = -2q + 70 \iff 3q = 45 \iff q = 15$$

and then $p = -15 + 35 = 20$. So the market equilibrium is for $p = 20$ and $q = 15$.

5. (20 points) The following laws for demand and supply are given for a certain type of production:

$$\begin{cases} pq = 12q + 15 \\ p = q + 10 \end{cases}$$

Find the market equilibrium.
Solution: We solve it by substitution. Using the second equation we substitute for $q + 10$ for p in the first equation to obtain:

$$(q + 10)q = 12q + 15 \iff q^2 + 10q = 12q + 15 \iff q^2 - 2q - 15 = 0$$

We solve this quadratic equation using the discriminant:

$$\Delta = 2^2 - 4(-15) = 4 + 60 = 64$$

Then we have 2 solutions

$$q_1 = \frac{2 + \sqrt{64}}{2} = 5 \text{ and } q_2 = \frac{2 - \sqrt{64}}{2} = -3$$

we keep only the first solution and then we get $p = 5 + 10 = 15$. So the market equilibrium is for $p = 15$ and $q = 5$.