

1. Draw the regions defined by the following inequalities (shade the correct region on the graph) and determine (exactly not graphically) their corners.
 a) $x \geq 0, y \geq 0, x + y \leq 3, 5x + 2y \leq 10$

See graph #1.

The “obvious” corners are $(0, 0), (0, 3), (2, 0)$ and the last one is found by solving the system:

$$\begin{cases} x + y = 3 \\ 5x + 2y = 10 \end{cases} \iff \begin{cases} y = 3 - x \\ 5x + 2(3 - x) = 10 \end{cases} \iff \begin{cases} y = 5/3 \\ x = 4/3 \end{cases}$$

then the last corner is $(4/3, 5/3)$.

- b) $x + y \geq 4, 3x + y \geq 6, x + 6y \geq 6$

See graph #2. The “obvious” corners are $(0, 6), (6, 0)$ another one is found by solving the system:

$$\begin{cases} x + y = 4 \\ 3x + y = 6 \end{cases} \iff \begin{cases} y = 4 - x \\ 3x + (4 - x) = 6 \end{cases} \iff \begin{cases} y = 3 \\ x = 1 \end{cases}$$

then the corner is $(1, 3)$. And the last corner is found by solving the system:

$$\begin{cases} x + y = 4 \\ x + 6y = 6 \end{cases} \iff \begin{cases} y = 4 - x \\ x + 6(4 - x) = 6 \end{cases} \iff \begin{cases} y = 2/5 \\ x = 18/5 \end{cases}$$

Then the last corner is $(18/5, 2/5)$.

2. a) Draw the feasible region defined by the following inequalities : $x \geq 0, y \geq 0, 5x + 3y \leq 15, x + 2y \leq 6$ (shade the correct region on the graph).

See graph #3

- b) Determine the corners of the feasible region (exactly, not by looking at the graph). The “obvious” corners are $(0, 0), (0, 3), (3, 0)$ the last one is found by solving the system:

$$\begin{cases} 5x + 3y = 15 \\ x + 2y = 6 \end{cases} \iff \begin{cases} 5(6 - 2y) + 3y = 15 \\ x = 6 - 2y \end{cases} \iff \begin{cases} 30 - 7y = 15 \\ x = 6 - 2y \end{cases} \iff \begin{cases} y = 15/7 \\ x = 12/7 \end{cases}$$

then the corner is $(12/7, 15/7)$.

- c) Find the maximum and the minimum of the function $f(x, y) = x + y$ and at which points they are achieved on the feasible region drawn in a).

The feasible region is closed and bounded, then maximum and minimum exist and are achieved on corners. We evaluate f at each corner: $f(0, 0) = 0, f(0, 3) = 3, f(3, 0) = 3$ and $f(12/7, 15/7) = 27/7 \approx 3.9$, then the maximum of f is $27/7$ and is achieved at the point $(12/7, 15/7)$ and the minimum of f is 0 and is achieved at the point $(0, 0)$.

3. Find the maximum value of $f(x, y) = 2x + y$ subject to the constraints:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + 2y \leq 40 \\ 3x + y \leq 60 \end{cases}$$

See graph #4. The “obvious” corners are $(0, 0), (0, 20), (20, 0)$. The last corner is obtained by solving

$$\begin{cases} x + 2y = 40 \\ 3x + y = 60 \end{cases} \iff \begin{cases} x = 40 - 2y \\ 120 - 6y + y = 60 \end{cases} \iff \begin{cases} x = 16 \\ y = 12 \end{cases}$$

Then the last corner is $(16, 12)$. We evaluate the function at each corner:

$$f(0, 0) = 0, f(0, 20) = 20, f(20, 0) = 40, f(16, 12) = 44$$

then the maximum value is 44.

4. Using the properties of logarithms, find the exact value of

a) $x = \log_4 2 + \log_4 32$

$$x = \log_4 2 + \log_4 32 = \log_4(2 * 32) = \log_4 64 = \log_4 4^3 = 3$$

b) $u = 4 \ln(e^2)$

$$u = 4 \ln e^2 = 4 * 2 \ln e = 8 \text{ because } \ln e = \log_e e = 1$$

c) $x = \log_6\left(\frac{1}{36}\right)$

$$x = \log_6\left(\frac{1}{36}\right) = \log_6(6^{-2}) = -2$$

5. Solve for x in

a) $\log_6(x^2 + 3x + 18) = 2$

$$\begin{aligned} \log_6(x^2 + 3x + 18) = 2 &\iff 6^{\log_6(x^2 + 3x + 18)} = 6^2 \iff x^2 + 3x + 18 = 36 \\ &\iff x^2 + 3x - 18 = 0 \end{aligned}$$

$\Delta = 9 - 4 * (-18) = 81 > 0$ so there are two solutions:

$$x_1 = \frac{-3 + \sqrt{81}}{2} = 3 \text{ and } x_2 = \frac{-3 - \sqrt{81}}{2} = -6$$

b) $50 + 10(1.02)^{2x} = 70$

$$\begin{aligned} 50 + 10(1.02)^{2x} = 70 &\iff 10(1.02)^{2x} = 20 \iff 1.02^{2x} = 2 \iff \ln(1.02^{2x}) = \ln 2 \\ &\iff 2x \ln(1.02) = \ln 2 \\ &\iff x = \frac{\ln 2}{2 \ln 1.02} \end{aligned}$$

6. You invest \$15,000 on an account at a rate 6% (yearly rate), compounded monthly. How long must you wait to have \$20,000 on this account?

Future value is given by $S = 15000(1.005)^t$, where t is the time in months.

You want to obtain $S = 20000$ so the equation is

$$\begin{aligned} 20000 = 15000(1.005)^t &\iff \frac{20000}{15000} = 1.005^t \iff 4/3 = 1.005^t \\ &\iff \ln(4/3) = t \ln(1.005) \\ &\iff t = \frac{\ln(4/3)}{\ln 1.005} \end{aligned}$$

then $t = 58$, that is you must wait 4 years and 10 months to achieve your goal.

7. The population of squirrel on a college campus in California is given by

$$y = \frac{200}{1 + 49e^{-0.25t}} \text{ where } t \text{ is in years}$$

- a) Find the number of squirrels after 1 year.

You plug $t = 1$:

$$y = \frac{200}{1 + 49e^{-0.25}} = 5$$

- b) When will the squirrel population reach 50?

You want to have $y = 50$ so the equation is

$$\begin{aligned} 50 = \frac{200}{1 + 49e^{-0.25t}} &\iff 50(1 + 49e^{-0.25t}) = 200 \iff 1 + 49e^{-0.25t} = 4 \\ &\iff 49e^{-0.25t} = 3 \\ &\iff e^{-0.25t} = 3/49 \\ &\iff -0.25t = \ln(3/49) \\ &\iff t = \frac{\ln(3/49)}{-0.25} = 11.2 \end{aligned}$$

You have to wait a bit more than 11 years.