

1. Solve the following linear systems of equations:

$$\text{a) } \begin{cases} x - 3y + 4z & = 0 \\ 2x + 2y + z & = 1 \\ 3x - 4y + 2z & = 9 \end{cases}$$

$$\text{b) } \begin{cases} x - y + z & = 3 \\ 3x + 2z & = 7 \\ x - 4y + 2z & = 5 \end{cases}$$

2. A bank lent a company \$135500 for the development of 3 new products, A, B and C. If the loan for product A is \$15500 more than for products B and C altogether, and the amount for product B was twice as much as the amount for product C, how much was lent for each product?

3. Find the inverse of the following matrices (if it exists):

$$\text{a) } \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

$$\text{b) } \begin{pmatrix} \frac{2}{3} & 2 \\ 2 & 6 \end{pmatrix}$$

$$\text{c) } \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

- 3.bis (you can use the result of exercise 3)

$$\text{a) Solve the following linear system: } \begin{cases} x + 2y & = 1 \\ 2x + 3y & = 45 \end{cases}.$$

$$\text{b) Let us define } A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}. \text{ Solve the following matrix equation}$$

$$\text{for } X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}:$$

$$AX = B$$

4. A small country with a simple economy (only agricultural and manufacturing industries) has the following technology matrix:

$$A = \begin{pmatrix} 0.5 & 0.5 \\ 0.1 & 0.3 \end{pmatrix}$$

- a) Calculate the matrix  $I_2 - A$  and find its inverse.  
 b) Using the correct Leontief model and the corresponding technological equation, find the gross production needed to have surpluses of 10 units of agricultural products and 25 of manufacturing products.

5. A company selling washers has fixed costs of \$100, and variable costs of  $100 + 0.2x$  per unit.
- Write the formula for the cost function  $C(x)$ .
  - If the selling price for a washer is \$112, calculate the profit function  $P(x)$  and find the break-even point(s).
  - How many washers must the company sell to maximize its profit?
6. Electricity bill:  $x$  is the consumption in kWh,  $C(x)$  is the monthly charge for  $x$  kWh. There is a fixed fee for the connection of \$10, and the price per kWh is of 10 cents when you consume less than 100 kWh, 8 cents when you consume between 100 and 200 kWh and 5 cents above 200 kWh. Write the piecewise defined function  $C(x)$ . How much do you pay if you consume 210 kWh?
7. Compute the following matrix operations, if possible. If not say why.

$$\begin{aligned} & \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \end{pmatrix} + 3 \begin{pmatrix} 0 & 1 & -1 \\ 1 & -2 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 2 & 4 & 6 \end{pmatrix} + (1 \ 2 \ 3 \ 4) \\ & \frac{1}{3} \begin{pmatrix} 1 & -1 & 0 \\ 5 & 6 & 9 \\ 8 & 7 & 12 \end{pmatrix} & \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix}^T - \begin{pmatrix} -1 & 1 \\ 2 & 0 \\ 3 & 1 \end{pmatrix} \end{aligned}$$