## MATH 1040

## Introduction to Statistics and Probability

# WORKSHEET - Extra examples <br> (Chapter 1: sections 1.1,1.2,1.3) 

## Chapter 1: Introduction to statistics

### 1.1 An Overview of Statistics

a) definitions of data, population, sample, population parameter, sample statistic;
b) difference between descriptive and inferential statistics

### 1.2 Data Classification

a) qualitative data vs. quantitative data;
b) levels of measurement (nominal, ordinal, interval, ratio)

### 1.3 Experimental Design

a) data collection (observational study, experiment, simulation, survey);
b) placebo effect, blinding technique, double-blind experiment, replication, randomization;
c) sampling techniques (census vs. random sample), random sampling (recognizing different sampling techniques: stratified, cluster, systematic, convenience)

## EXTRA EXAMPLES

1. Identify the population and the sample:
a) A survey of 1353 American households found that $18 \%$ of the households own a computer.
b) A recent survey of 2625 elementary school children found that $28 \%$ of the children could be classified obese.
c) The average weight of every sixth person entering the mall within 3 hour period was 146 lb .
2. Determine whether the numerical value is a parameter or a statistics (and explain):
a) A recent survey by the alumni of a major university indicated that the average salary of 10,000 of its 300,000 graduates was 125,000 .
b) The average salary of all assembly-line employees at a certain car manufacturer is $\$ 33,000$.
c) The average late fee for 360 credit card holders was found to be $\$ 56.75$.
3. For the studies described, identify the population, sample, population parameters, and sample statistics:
a) In a USA Today Internet poll, readers responded voluntarily to the question "Do you consume at least one caffeinated beverage every day?"
b) Astronomers typically determine the distance to galaxy (a galaxy is a huge collection of billions of stars) by measuring the distances to just a few stars within it and taking the mean (average) of these distance measurements.
4. Identify whether the statement describes inferential statistics or descriptive statistics:
a) The average age of the students in a statistics class is 21 years.
b) The chances of winning the California Lottery are one chance in twenty-two million.
c) There is a relationship between smoking cigarettes and getting emphysema.
d) From past figures, it is predicted that $39 \%$ of the registered voters in California will vote in the June primary.
5. Determine whether the data are qualitative or quantitative:
a) the colors of automobiles on a used car lot
b) the numbers on the shirts of a girl's soccer team
c) the number of seats in a movie theater
d) a list of house numbers on your street
e) the ages of a sample of 350 employees of a large hospital
6. Identify the data set's level of measurement (nominal, ordinal, interval, ratio):
a) hair color of women on a high school tennis team
b) numbers on the shirts of a girl's soccer team
c) ages of students in a statistics class
d) temperatures of 22 selected refrigerators
e) number of milligrams of tar in 28 cigarettes
f) number of pages in your statistics book
g) marriage status of the faculty at the local community college
h) list of 1247 social security numbers
i) the ratings of a movie ranging from "poor" to "good" to "excellent"
j) the final grades (A,B,C,D, and F) for students in a chemistry class
k) the annual salaries for all teachers in Utah
1) list of zip codes for Chicago
$\mathrm{m})$ the nationalities listed in a recent survey
n) the amount of fat (in grams) in 44 cookies
o) the data listed on the horizontal axis in the graph

7. Decide which method of data collection you would use to collect data for the study (observational study, experiment, simulation, or survey):
a) A study of the salaries of college professors in a particular state
b) A study where a political pollster wishes to determine if his candidate is leading in the polls
c) A study where you would like to determine the chance getting three girls in a family of three children
d) A study of the effects of a fertilizer on a soybean crop
e) A study of the effect of koalas on Florida ecosystem
8. Identify the sampling technique used (random, cluster, stratified, convenience, systematic):
a) Every fifth person boarding a plane is searched thoroughly.
b) At a local community College, five math classes are randomly selected out of 20 and all of the students from each class are interviewed.
c) A researcher randomly selects and interviews fifty male and fifty female teachers.
d) A researcher for an airline interviews all of the passengers on five randomly selected flights.
e) Based on 12,500 responses from 42,000 surveys sent to its alumni, a major university estimated that the annual salary of its alumni was 92,500 .
f) A community college student interviews everyone in a biology class to determine the percentage of students that own a car.
g) A market researcher randomly selects 200 drivers under 35 years of age and 100 drivers over 35 years of age.
h) All of the teachers from 85 randomly selected nation's middle schools were interviewed.
i) To avoid working late, the quality control manager inspects the last 10 items produced that day.
j) The names of 70 contestants are written on 70 cards, The cards are placed in a bag, and three names are picked from the bag.
9. Explain what bias there is in a study done entirely online.
10. A local newspaper ran a survey by asking, "Do you support the development of a weapon that could kill millions of innocent people?" Determine whether the survey questions is biased and why.

## SOLUTIONS:

1. a) population: all American households sample: collection of 1353 American households surveyed
b) population: all elementary school children
sample: collection of 2625 elementary school children surveyed
c) population: all people entering the mall within the assigned 3 hour period sample: every $6^{\text {th }}$ person entering the mall within the 3 hour period
2. a) statistic - part of 300,000 graduates are surveyed
b) parameter - all assembly-line employees were included in the study
c) statistic - 360 credit cards were examined (not all)
3. a) population: all readers of USA Today; sample: volunteers that responded to the survey; population parameter: percent who have at least one caffeinated drink among all readers of USA Today; sample statistic: percent who have at least one caffeinated drink among those who responded to the survey
b) population: all starts in the galaxy; sample: the few stars selected for measurements; population parameter: mean (average) of distances between all stars and Earth; sample statistics: mean of distances between the stars in the sample and Earth
4. a) descriptive
b) inferential
c) inferential
d) inferential
5. a) qualitative
b) qualitative
c) quantitative
d) qualitative
e) quantitative
6. a) nominal
b) nominal
c) ratio
d) interval
e) ratio
f) ratio
g) nominal
h) nominal
I) ordinal
j) ordinal
k) ratio
l) nominal
m) nominal
n) ratio
o) ratio
7. systematic
cluster
stratified
cluster
random
convenience
stratified
cluster
convenience
random
8. a) survey
b) observation
c) simulation
d) experiment
e) simulation
9. It is limited to people with computers.
10. Yes - it tends to encourage negative responses.

## Chapter 2: Descriptive Statistics

### 2.1 Frequency Distributions and Their Graphs

a) organizing and describing data sets using frequency distribution table (finding the class width, constructing a frequency distribution table)
b) graphs: frequency histogram, frequency polygon, relative frequency histogram, ogive- cumulative frequency graph

### 2.2 More Graphs and Displays

a) display of quantitative data (stem-and-leaf plot, dot plot)
b) display of qualitative data (pie chart, Pareto chart)
c) display of paired data sets (scatter plot, time series chart)

### 2.3 Measures of Central Tendency

a) typical/central entry of a data set (mean, mode, median)
b) weighted mean and the mean of a frequency distribution (using classes and class midpoints)
c) interpreting the shape of distribution (symmetric, uniform, skewed left/right)

### 2.4 Measures of Variation

a) range
b) deviation (from the mean of the data set)
c) variance and standard deviation (how to find these values)
d) interpreting standard deviation using Empirical Rule (68-95-99.7 rule) for the bell-shaped distribution
e) interpreting standard deviation in connection with not bell-shaped distributions using Chebychev's Theorem
f) standard deviation for grouped data (using classes and midpoints)
g) CV - coefficient of variation of a data set

### 2.5 Measures of Position

a) finding and interpreting the meaning of quartiles using different data sets
b) interquartile range and outliers
c) five-number summary (Box-and-Whisker plot)
d) finding and interpreting percentiles
e) finding the $z$-score (the standard score) and comparing $z$-scores from different data sets

### 2.1 Frequency Distributions and Their Graphs

Example 1:
The following data set lists the midterm scores received by 50 students in a chemistry class:
$\begin{array}{llllllllll}45 & 85 & 92 & 99 & 37 & 68 & 67 & 78 & 81 & 25\end{array}$
$\begin{array}{llllllllll}97 & 100 & 82 & 49 & 54 & 78 & 89 & 71 & 94 & 87\end{array}$

| 21 | 77 | 81 | 83 | 98 | 97 | 74 | 81 | 39 | 77 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$\begin{array}{llllllllll}99 & 85 & 85 & 64 & 92 & 83 & 100 & 74 & 68 & 72\end{array}$
$\begin{array}{llllllllll}65 & 84 & 89 & 72 & 61 & 49 & 56 & 97 & 92 & 82\end{array}$
Construct a frequency distribution, frequency histogram, relative frequency histogram, frequency polygon, and cumulative frequency graph (ogive) using 6 classes.

## Example 2:

The heights (in inches) of 30 adult males are listed below.
$\begin{array}{llllllllll}70 & 72 & 71 & 70 & 69 & 73 & 69 & 68 & 70 & 71\end{array}$
$\begin{array}{llllllllll}67 & 71 & 70 & 74 & 69 & 68 & 71 & 71 & 71 & 72\end{array}$
$\begin{array}{llllllllll}69 & 71 & 68 & 67 & 73 & 74 & 70 & 71 & 69 & 68\end{array}$
Construct a frequency distribution, frequency histogram, relative frequency histogram, frequency polygon, and cumulative frequency graph (ogive) using 5 classes.

### 2.2 More Graphs and Displays

Example 1:
Weights of Carry-On Luggage in Pounds
(for a random sample of 40 passengers returning from a vacation to Hawaii)
$\begin{array}{llllllllll}30 & 27 & 12 & 42 & 35 & 47 & 38 & 36 & 27 & 35\end{array}$
$\begin{array}{llllllllll}22 & 29 & 17 & 3 & 21 & 0 & 38 & 32 & 41 & 33\end{array}$
$\begin{array}{llllllllll}26 & 45 & 18 & 43 & 18 & 32 & 31 & 32 & 19 & 21\end{array}$
$\begin{array}{llllllllll}33 & 31 & 28 & 29 & 51 & 12 & 14 & 21 & 18 & 26\end{array}$

## Example 2:

The following table represents a recent survey of 500 people that answered the following question: "How long do you spend on home telephone after 5 pm ?"

| Time | Number |
| :--- | :---: |
| Less than $1 / 2 \mathrm{hr}$. | 296 |
| $1 / 2 \mathrm{hr}$ to 1 hr | 83 |
| more than 1 hr | 121 |

Example 3:
What will most likely cause you to be late to your early morning class?

| Cause | Frequency |
| :--- | :---: |
| snoozing after alarm | 20 |
| goes off |  |
| car trouble | 3 |
| too long over breakfast | 15 |
| last minute studying | 18 |
| finding the right clothes | 12 |
| other | 4 |

Example 4: The following table gives mean daily caloric intake (all residents) and infant mortality rate (per 1,000 births) for nine countries:

| Country | Mean daily calories | Infant mortality rate |
| :--- | :---: | :---: |
| Afganistan | 1523 | 154 |
| Austria | 3495 | 6 |
| Colombia | 2678 | 24 |
| Ehtiopia | 1610 | 107 |
| Germany | 3443 | 6 |
| Liberia | 1640 | 153 |
| New Zealand | 3362 | 7 |
| Turkey | 3429 | 44 |
| U.S. | 3671 | 7 |

Example 5: You have been in the walking/jogging exercise program for 20 weeks, and for each week you have recorded the distance (in miles) you covered in 30 minutes.

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance | 1.5 | 1.4 | 1.7 | 1.6 | 1.9 | 2.0 | 1.8 | 2.0 | 1.9 | 2.0 |


| Week | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Distance | 2.1 | 2.1 | 2.3 | 2.3 | 2.2 | 2.4 | 2.5 | 2.6 | 2.4 | 2.7 |

### 2.3 Measures of Central Tendency

Example 1:
The top 14 speeds, in mph, for Pro-Stock drag racing over the past two decades are:
$\begin{array}{lllllll}181.1 & 202.2 & 190.1 & 201.4 & 191.3 & 201.4 & 192.2\end{array}$
$\begin{array}{lllllll}201.2 & 193.2 & 201.2 & 194.5 & 199.2 & 196.0 & 196.2\end{array}$
Example 2:
The scores of the top ten finishers in a recent golf tournament:
$\begin{array}{llllllllll}71 & 67 & 67 & 72 & 76 & 72 & 73 & 68 & 72 & 72\end{array}$

## Example 3:

The ages of 21 cars randomly selected in a student parking lot:
$\begin{array}{lllllllllllllllllllll}12 & 6 & 4 & 9 & 11 & 1 & 7 & 8 & 9 & 8 & 9 & 13 & 5 & 15 & 7 & 6 & 8 & 8 & 2 & 1 & 5\end{array}$
Example 4:
You are taking a class in which your grade is determined from:
Quizzes: 15\% total
Exams (3): $15 \%$ each
Homework: 10\%
Final: 30\%
Your scores are: average quiz mean $=85$, exams $=78,81,92$, homework mean $=85$ and your final $=89$.
a) If the minimum average for $\mathrm{B}+$ is 87 , did you get $\mathrm{B}+$ at the end of the semester?
b) What if the quiz mean was not given, but the quiz scores are given as $10,12,8,2,9,7$ (out of 12 points each), and only best 4 quizzes count towards your grade?

Example 5:
The average starting salary for 50 employees at a company is given:
10 with high-school diploma: \$27,000 each
25 with BA/BS degree: $\$ 35,000$ each
10 with MA/MS degree: $\quad \$ 47,000$ each
5 with Ph.D. degree: \$59,500 each
How would you find the starting mean salary for these employees?
Would you add the $4 \$$ amounts and divide by 4 ? Explain

## Example 6:

Approximate the mean of the grouped data:

| Heights of Males (inches) | Frequency |
| :---: | :---: |
| $63-65$ | 3 |
| $66-68$ | 6 |
| $69-71$ | 7 |
| $72-74$ | 4 |
| $75-77$ | 3 |

### 2.4 Measures of Variation

## Example 1:

Sample annual salaries (in thousand dollars) for public elementary school teachers are listed:
$\begin{array}{lllllll}26.0 & 19.9 & 17.8 & 31.4 & 38.6 & 28.7 & 25.0\end{array}$

Example 2:
The heights (in inches) of ALL 10 adult males in an office are:
$\begin{array}{llllllllll}70 & 72 & 71 & 70 & 69 & 73 & 69 & 68 & 70 & 71\end{array}$
Example 3:
Midterm scores for 20 random students (in a class of 100):
$\begin{array}{llllllllll}82 & 45 & 37 & 98 & 100 & 74 & 87 & 89 & 63 & 76\end{array}$
$\begin{array}{llllllllll}75 & 61 & 43 & 99 & 86 & 86 & 75 & 92 & 65 & 80\end{array}$
What is the meaning of s (sample standard deviation)?

## Example 4:

The mean value of land and buildings per acre from a sample of farms is $\$ 2,400$, with a standard deviation of $\$ 450$. Assuming that the prices are normally distributed,
a) which of the following prices ( $\$ /$ acre) would be considered unusual? $3,325 \quad 2,450 \quad 3,200 \quad 1,490 \quad 1,675 \quad 2,950$
b) Find the percentage of farms that are more than $\$ 3,300$ per acre.
c) Find the percentage of farms that are priced between $\$ 1950$ and $\$ 3,750$ per acre.

Example 5:
How long will it take you to graduate from college? (These are responses from a sample of students on campus.)

| Years | f |
| :--- | :---: |
| 3 | 10 |
| 4 | 48 |
| 5 | 25 |
| 6 | 10 |
| 7 | 6 |
| 8 | 1 |

Assuming the bell-shaped distribution (normal distribution):
What percentage of students will need:
a) more than 7.9 years to graduate?
b) between 3.5 and 5.7 years to graduate?
c) more than 1.3 years to graduate?
d) between 2.4 and 5.7 years to graduate?
e) between 5.7 and 7.9 years to graduate?
f) between 4.6 and 5.7 years to graduate?

## Example 6:

A random sample of the age of employees in a City Hall:

| Age | frequency |
| :--- | :---: |
| $20-29$ | 5 |
| $30-39$ | 10 |
| $40-49$ | 12 |
| $50-59$ | 8 |
| $60-69$ | 5 |

Assuming that the ages are normally distributed:
a) what percentage of the City Hall employees are between 31.8 and 68.4 years old?

If there are 120 employees in a City Hall, approximately how many of them are:
b) between 31.8 and 56.2 years old?
c) older than 68.4 ?

## Example 7:

The mean time in a women's $400-\mathrm{m}$ dash is 57.07 s , with a standard deviation of 1.05 s .
a) Apply Chebychev's Theorem to the data using $\mathrm{k}=3$. Explain the meaning of the values that you find.
b) If there is a sample of 350 women, about how many would have time between 54.97 s and 59.17 s?

## Example 8:

In a random sample, 10 students were asked to compute the distance they travel one way to school to the nearest tenth of a mile. The data is listed:
$\begin{array}{llllllllll}1.1 & 5.2 & 3.6 & 5.0 & 4.8 & 1.8 & 2.2 & 5.2 & 1.5 & 0.8\end{array}$
a) Using Chebychev's Theorem, approximate the percentage of students travel between 0 and 6.76 miles (one way) to school.
b) If there are 25,000 students on campus, approximate the number of students who travel between 0 and 6.76 miles (one way) to school.

## Example 9:

SAT verbal scores are normally distributed with a mean of 489 and a standard deviation of 93. Use the Empirical Rule (also called 68-95-99.7 Rule) to determine what percentage of the scores lie:
a) between 303 and 582 .
b) above 675 ?
c) If 3,500 students took the SAT verbal test, about how many received between 396 and 675 points

Example 10:
Find the coefficient of variation for the data set given in examples 5 and 6.

## Example 11:

The batting averages of Sammy Sosa and Barry Bonds for 13 recent years:
Sosa: $\quad \bar{x}=0.279, \mathrm{~s}=0.033$
Bonds: $\bar{x}=0.312, \mathrm{~s}=0.027$
Which player is more consistent? Why?
Example 12:
Which data set has the highest a) mean , b) standard deviation

i) | 0 | 9 |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | 1 | 5 | 8 |  |
|  |  |  |  |  |
| 2 | 3 | 3 | 7 | 7 |
|  | 3 | 2 | 5 |  |
|  | 4 | 1 |  |  |
|  |  |  |  |  |

ii) 0
1 589
$2 \begin{array}{lllll}2 & 3 & 7 & 3\end{array}$
3256
iii) $10 \quad 9$
1158
$12 \quad 3377$
1325
$14 \quad 1$

## Example 13:

Data entries: a blcced Mean of a, b, c, d is $\bar{x}$, and the standard deviation is s. What will happen to the mean and standard deviation if we add 5 to each data entry?
What will happen to the mean and standard deviation if each data entry is 3 times larger.

### 2.5 Measures of Position

## Example 1:

The ACT math scores of 15 high-school seniors:
$\begin{array}{lllllllllllllll}18 & 15 & 25 & 24 & 21 & 17 & 32 & 30 & 20 & 27 & 28 & 24 & 18 & 27 & 28\end{array}$

## Example 2:

In a random sample, 18 students were asked how far away (in miles) from campus do they live.

| 4.2 | 3.7 | 15.4 | 12.7 | 0.5 | 0 | 4.3 | 1.6 | 13.8 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 14.0 | 11.3 | 8.2 | 4.0 | 12.9 | 20.5 | 18 | 7.5 | 2.1 |

## Example 3:

The heights (in inches) of 20 women are listed below.

| 70 | 71 | 65 | 60 | 62 | 68 | 67 | 68 | 70 | 73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 60 | 61 | 61 | 65 | 64 | 68 | 69 | 69 | 63 | 65 |


| 60 | 61 | 61 | 65 | 64 | 68 | 69 | 69 | 63 | 65 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

a) Find the height that corresponds to $65^{\text {th }}$ percentile.
b) What percentile corresponds to the height of 70 ?
c) Find the height that corresponds to $27^{\text {th }}$ percentile?

## Example 4:

| 10 | 493 |
| :--- | :--- | :--- |

$11 \quad 5983$
$12 \quad 39776$
13265
14063
a) Find the score that corresponds to 44th percentile?
b) What percentile corresponds to the score of 135?
c) What is the score that corresponds to $11^{\text {th }}$ percentile?

## Example 5:

John is taking biology and math. On the biology exam the mean is 82 with a standard deviation of 6 , and on the math exam the mean is 78 with a standard deviation of 3 . John received 85 on the biology exam, 80 on the math exam.

Use z-scores to determine on which test he performed better. What would you say if John scored 92 on the math exam? What would you say if John received 79 on the biology exam and 76 on the math exam?

## Chapter 3: Probability

### 3.1 Basic Concepts of Probability and Counting

a) defining terms: probability experiment, outcome, sample space, event, simple event
b) The Fundamental Counting Principle and lots of examples using the principle
c) Identifying 3 types of probability (classical/theoretical, empirical/statistical, and subjective) and finding probabilities for all three types
d) Law of large numbers
e) Complement of event E
f) illustrating the concept using variety of examples drawn from different fields of study

### 3.2 Conditional Probability and the Multiplication Rule

a) conditional probability
b) independent vs. dependent events and their probabilities
c) the multiplication rule for $\mathrm{P}(\mathrm{A}$ and B$)$ for dependent and independent events; extending this rule for 3 or more events
d) working on a variety of examples to illustrate the multiplication rule

### 3.3 The Addition Rule

a) defining mutually exclusive events
b) finding the probability $\mathrm{P}(\mathrm{A}$ or B$)$ if A and B are (or not) mutually exclusive events
c) working on a variety of examples to illustrate the addition rule

### 3.4 Additional Topics in Probability and Counting

a) ordered arrangements (permutations)
b) permutations of $n$ objects taken $t$ at a time (the order does matter)
c) combinations of $n$ objects taken $r$ at a time (the order does not matter)
d) finding probabilities using the counting principle, permutations and combinations

CARDS: standard deck of cards has 52 cards.
13 of each: hearts, diamonds, clubs and spades
red cards: hearts and diamonds (total 26)
black cards: clubs and spades (total 26)
4 of each: $2,3,4,5,6,7,8,9,10$, A, J, Q, K
face cards: J, Q, K (12 total)

### 3.1 Basic Concepts of Probability and Counting

1. You have a red and black six-sided dice.
a) Develop the sample space that results from rolling these two dice.
b) Find all possible outcomes of getting at least 5 with the black die.
c) Is "getting the sum of 4 " a simple event?
d) Is "getting the sum of 2 " a simple event? "getting an even number on both dice"?
2.a) How many pairs of letters from the English alphabet (with replacement) are possible? (Disregard the difference between uppercase and lower case letters.)
b) How will your answer change if replacements are not allowed?
c) What if we make a difference between the uppercase and lowercase letters (without replacement)?
2. The phone numbers in U.S. consist of 10 digits ( 3 digits area code +7 digits local number). How many different telephone numbers are possible within each area code, assuming that the local number cannot begin with 0 or 1 ?
3. Restaurant menu has: 5 appetizers, 10 main dishes, 4 desserts, 5 drinks. If you would like to order all 4 , how many different meals can you order?
4. Utah license plate contains 6 characters: 1 letter followed by 3 numbers, followed by 2 letters (letters and numbers can repeat).
a) How many different license plates can the state of Utah issue?
b) What happens if letters and numbers cannot be repeated?
6.2 coins (a dime and a nickel) are tossed. Find the probability that a) both coins land heads up, b) you get 1 head and 1 tail ?
5. A card is drawn from a standard deck of playing cards. Find the probability:
a) the card drawn is an ace,
b) the card drawn is a diamond,
c) the card drawn is a diamond, a heart, or a club.
6. Two 6 -sided dice (black and white) are tossed. What is the probability that:
a) the sum of the two dice is 8 ?
b) the sum is 14 ?
c) The sum is at least 6?
d) The sum is not 7?
7. The probability that a manufactured part for the computer is working is 0.992 . What is the probability that the part is not working?
8. How long does it take you to get ready for work/school:

| Response | frequency |
| :--- | :--- |
| $0-20 \mathrm{~min}$ | 25 |
| $20-40 \mathrm{~min}$ | 75 |
| $40-60 \mathrm{~min}$ | 37 |
| more than 1 hr | 15 |

What is the probability that the next person that answers to the survey says that it takes $\mathrm{him} / \mathrm{her} \quad$ a) $40-60 \mathrm{~min}$. to get ready? $\quad$ b) $20-40 \mathrm{~min}$. or $40-60 \mathrm{~min}$. to get ready?
11. The surgeon tells you that for every 150 surgeries that he/she performs, 6 patients need to come back for the second surgery. If you are the next patient, find the probability that you would need to have the second surgery.
12. Assume that the probability of having a boy or a girl is 0.5 . In a family of 5 children, what is the probability that:
a) all children are boys,
b) all the children are the same gender,
c) there is at least 1 girl.
13. A probability experiment consists of tossing a coin and rolling a six-sided die.
a) Draw a tree diagram.

Find the following probabilities:
b) tossing a tail and rolling an even number,
c) tossing a head or tail and rolling a number greater than 3 ,
d) tossing a head or rolling a number greater than 3 .
e) Find the complement of the event in c) and d).
14. The heights (in inches) of all males enrolled in history class:
$6 \quad 5566689999$
$7 \quad 00122234455667$
If a male student is selected at random, find the probability that his height is:
a) at least 69 in .
b) between 70 in . and 73 in . (inclusive)
c) more than 75 in .
d) not 69 in .

### 3.2 Conditional Probability and the Multiplication Rule

1. Two cards are selected in sequence from a standard deck ( 52 cards). Find the probability that the second card is an ace, given that the first card is:
a) a seven (assume that it is not replaced),
b) an ace (assume that it is not replaced),

What would be the answer in $a$ ) and $b$ ) if the first card is replaced?
2. Study of the effect of coffee on gall stones:

|  | Gall stone disease | No disease |
| :--- | :--- | :---: |
| No coffee | 385 | 14,068 |
| Coffee | 91 | 4,806 |

Find the probability that a patient:
a) has a gall stone disease, given that he/she does not consume coffee,
b) does not have a gall stone disease, given that he/she does consume coffee.
3. You are spinning the spinner with numbers 1 through 6 . Assume that the spinner cannot stop on a line. You are spinning twice. Find the probability that:
a) the first spin stops on an odd number and the second spin stops on a multiple of 3 ,
b) the first spin shows a number less than 4 and the sum of 2 spins is equal to 5 .
4. The probability that a student will pass organic chemistry class is 0.85 .

3 students are randomly selected. Find the probability that:
a) all 3 will pass the class,
b) none of 3 students will pass the class,
c) at least 1 student will pass.
5. The probability that a person in the U.S. has type A+ blood is $32.5 \%$. Five unrelated people in the U.S. are selected at random. Find the probability that:
a) all five have type A+.
b) none of the five has type A+.
c) at least one of the five has type $\mathrm{A}+$.
6. If you roll a 6 sided die 8 times, find the probability that you roll:
a) an odd number at least once.
b) a 5 at least once.
7. You purchase 20 lottery tickets for which the probability of winning some prize on a single ticket is 1 in 10 . What is the probability that you have at least 1 winning ticket among the 20 tickets?
8. Find the probability of drawing 3 diamonds in a row from a regular deck of cards if:
a) the drawn card is returned to the deck each time,
b) the drawn card is not returned to the deck each time.
9. Refer to problem \#8. Find the probabilities if you are drawing 3 fives in a row (with replacements and without replacements).
10. A 3 person jury must be selected at random (without replacement) from a pool of 12 people that has 7 men and 5 women. Find the probability of selecting an a) all-female jury, b) all-male jury.

11. Student ages | Frequency |  |
| :---: | :---: |
| $17-26$ | 149 |
| $27-36$ | 85 |
| $37-46$ | 46 |
| $47-56$ | 15 |
| 57 and over | 5 |

a) Find the probability that a student chosen at random is between 27 and 36 years old.
b) If four students are randomly selected (without replacement), find the probability that all four students are between 37 and 46 years old.
c) If four students are randomly selected (without replacement), what is the probability that at least 1 will be 57 years or older?
d) If four students are randomly selected (without replacement), what is the probability that none of these four students are between 17 and 26 years old?
12. Refer to the problem \#14 from 3.1 and assume that there are no replacements.
a) Find the probability that 2 randomly chosen male students are both between 69 in . and 73 in. tall.
b) Find the probability that at least 1 of 2 students is between 69 in. and 73 in. tall.
13. The access code for a garage door consists of 5 digits. The first digit cannot be 0 . Find the probability that you guess the code from the first try?
14. Of campus professors $60 \%$ are male, and of these, $15 \%$ work for College of Humanities. Find the following probabilities:
a) randomly selected professor is a male and works for College of Humanities.
b) randomly selected professor is a male and does not work for College of Humanities.
c) randomly selected professor is a female and works for College of Humanities.
15. The following graph shows the types of incidents encountered with drivers using cell phones.

a) Find the probability that a randomly chosen incident involves cutting off a car.
b) Find the probability that two randomly chosen incidents (without replacement) both had an accident.
c) Find the probability that a randomly chosen incident did not involve cutting off a car.
d) Find the probability that from randomly selected 3 incidents (without replacement) at least one involved speeding up.
16. Three people are selected at random. Find the probability:
a) all 3 are born on Tuesday
b) all 3 are born on the same day of the week
c) all 3 are born on a different day of the week

### 3.3 The Addition Rule

1. You are rolling a single die. Find the probability of rolling:
a) either a 2 or a 5 ,
b) either a 2 or an odd number,
c) either a 4 or an even number.
2. You have a standard deck of cards. Find the probability of drawing:
a) either a queen or a spade,
b) either a jack or a king,
c) either a queen or a red card,
d) either a king or a face card.
3. You select one person at random from a room with 15 people: 5 blond men, 2 dark hair men, 4 blond women and 4 dark hair women. What is the probability that you will select:
a) either a woman or a blond person,
b) a man,
c) either a man or a dark hair person?
4. If you roll a die, find the probability that you roll a number greater than 4 or an odd number?
5. A card is selected at random from a standard deck of cards. Find each probability:
a) Randomly selected card is a black suit or a king.
b) Randomly selected card is a four or a face card.
c) Randomly selected card is a 7 or a king.
d) Randomly selected card is a club or a face card
6. 

|  | Psychology major | Non-Psychology major |
| :--- | :---: | :---: |
| Males | 150 | 9,750 |
| Females | 375 | 11,300 |

A student is selected at random. Find the following probabilities:
a) the student is a female or a psychology major
b) the student is a male or not psychology major
c) the student is a male and psychology major
d) Given that the student is a female, what is the probability that she is a psychology major?
e) Given that the student is a psychology major, find the probability that the student is a female.
f) If we select 3 students at random (without replacement), what is the probability that all 3 students are women and majoring in psychology? What is the probability if we are using replacement?
g) If we select 3 students at random (with replacement) find the probability that at least one is a male who is not majoring in psychology. Find the probability if the replacement is not used.
7. Sam is playing a game with 2 dice (black and red) and on his next move he needs to throw a sum bigger than 9 in order to win. What is the probability that Sam will win?
8. If you roll a die, find the probability that you roll a number less than 5 or an even number.
9. If we have 3 sets $\mathrm{A}, \mathrm{B}$ and C , find P ( A or B or C ).
10. From Section 2.2. we had this Pareto chart:


If you randomly selected a person from a sample, find each probability:
a) The person is late because of last minute studying or clothes trouble.
b) The person is not late because of last minute studying.
c) You randomly selected 4 people from the study i) without replacement, ii) with replacement. What is the probability that all 4 were late because of car trouble?
d) You randomly selected 4 people from the study i) without replacement, ii) with replacement. Find the probability that all 4 were late because of trouble with clothes. e) If you randomly select 4 people form the study i) with replacement, ii) without replacement, find the probability that at least one was late because of snoozing.

### 3.4 Additional Topics in Probability and Counting

1. In how many different ways can we arrange letters $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ ?
2. We have 4 objects, $A, B, C$, and $D$, and we want to make ordered arrangements of 2 objects. How many would we have?
3. How many distinguishable permutations can you make out of letters that make the word a) MATHEMATICS
b) STATISTICS ?
4. Suppose you coach a team of 12 swimmers and you need to put together a 4-person relay team. In how many different ways can you do this?
5. Calculate:

| $15!=$ | $\frac{{ }_{10} C_{3}}{{ }_{40} C_{3}}=$ |  |
| :--- | :--- | :--- |
| $12!=$ | $\frac{15!}{12!}=$ |  |
| ${ }_{10} C_{3}=$ | ${ }_{40} C_{3}=$ | $\frac{18!}{13!}=$ |

6. The scholarship committee is considering 25 applicants for 3 awards ( $1^{\text {st }}$ award $\$ 3,500,2^{\text {nd }}$ award $-\$ 3,000,3^{\text {rd }}$ award $-\$ 2,000$ ).
a) How many different ways are possible to award these scholarships?
b) Would you change anything about the solution if all three awards were $\$ 2,500$ ?
7. There are 30 passengers that still need to check-in and get a boarding pass. The airline representative will upgrade 5 passengers to the first class, seats $1 \mathrm{~B}, 1 \mathrm{D}, 3 \mathrm{~A}, 3 \mathrm{C}, 4 \mathrm{~B}$.
In how many different ways can the airline representative do this?
8. 20 runners enter the competition. In how many ways can they finish $1^{\text {st }}, 2^{\text {nd }}$, and $3^{\text {rd }}$ ?
9. In how many ways can you pick a group of 4 volunteers from 35 students?
10. How many ways can 3 Republicans, 2 Democrats, and 1 Independent be chosen from 10 Republicans, 8 Democrats, and 5 Independents to fill 6 positions on City Council? Find the probability that this will happen.
11. A security code consists of 2 letters followed by 3 digits. The first letter can not be A, B , or C , and the last digit can not be a 0 . What is the probability of guessing the security code in one trial?
12. A shipment of 40 fancy calculators contains 5 defective units. In how many ways can a college bookstore buy 20 of these units and receive:
a) no defective units
b) one defective unit
c) at least 17 good units
d) What is the probability of the bookstore receiving 2 defective units?
e) Find the probability of receiving at most 2 bad calculators.
f) Find the probability of receiving at least 4 defective units.
13. You are dealt a hand of four cards from a standard deck. Find the probability that:
a) the first three cards are of the same suit and one is of a different suit.
b) three cards are of the same suit and one is of a different suit.

## Chapter 4: Probability Distributions

### 4.1 Probability Distributions

a) random variable (discrete vs. continuous)
b) discrete probability distribution table
c) mean, variance, and standard deviation of a discrete random variable
d) expected value of a discrete random variable (mean)

### 4.2 Binomial Distributions

a) binomial experiment
b) binomial probability formula
c) constructing a binomial distribution

### 4.1 Probability Distributions

1. Doctor's office collected the results of 350 patients who answered the following question:" Rate your pain from 1 to 10 , where 1 -almost no pain, 10-extreme pain". These were the results:

| Rating, x | Frequency |
| :---: | :---: |
| 1 | 22 |
| 2 | 27 |
| 3 | 35 |
| 4 | 36 |
| 5 | 40 |
| 6 | 56 |
| 7 | 70 |
| 8 | 31 |
| 9 | 13 |
| 10 | 20 |

Find the mean.
2. Construct a probability distribution:

| number of <br> cars, x | households | $\mathrm{P}(\mathrm{x})$ |
| :--- | :---: | :---: |
|  |  |  |
| 0 | 100 |  |
| 1 | 325 |  |
| 2 | 450 |  |
| 3 | 94 |  |
| 4 | 31 |  |

Find the probability of randomly selecting a household that has
a) 2 cars,
b) at least two cars,
c) at most three cars.
d) Find the mean (expected value).
3. Local charity is selling $\$ 5$ raffle tickets as part of Christmas fund-raising program.

1,200 tickets are sold. The four prizes are $\$ 1,000, \$ 800, \$ 500$, and $\$ 350$.
You buy one ticket. What is $\mathrm{E}(\mathrm{x})$, expected value of your gain/loss?
4. 1,500 students were asked:" How many parking tickets did you receive within the last 5 years?"

| number of <br> parking tickets <br> x | $\mathrm{P}(\mathrm{x})$ |  |
| :--- | :--- | :--- |
| 0 | 0.150 |  |
| 1 | 0.280 |  |
| 2 | 0.267 |  |
| 3 | 0.167 |  |
| 4 | 0.100 |  |
| 5 | 0.036 |  |
|  |  |  |

Find the expected value / mean and standard deviation for the number of parking tickets.
5. The total number of defects per batch of DVDs inspected is given:

| Defects | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Batches | 95 | 113 | 87 | 64 | 13 | 8 |

Construct a probability distribution and find the mean and standard deviation for the number of defects per batch using the probability distribution.

### 4.2 Binomial Distributions

1. Decide whether the experiment is a binomial experiment. If it is, specify the values of $\mathrm{n}, \mathrm{p}, \mathrm{q}$, and x . If it is not, explain why.
a) A bag contains 4 white marbles, 7 red marbles, and 5 black marbles. You randomly select 3 marbles from the bag, without replacement. The random variable represents the number of black marbles.
b) A certain medical procedure has $85 \%$ chance of success on patients. The surgery is performed on 8 patients. The random variable represents the number of successful surgeries.
c) You take an exam that consists of 10 multiple-choice questions. Each question has 5 possible answers, only one of which is correct. You will randomly guess the answer to each question. The random variable represents the number of correct answers.
2. Refer to 1 b ). Create a binomial probability distribution. Then, find the probability of the surgery being successful on:
a) (exactly) 6 patients,
b) at least 5 patients,
c) at most 7 patients.
3. Refer to 1c).
a) Create a binomial probability distribution.
b) To get at least a B, you need to answer at least 8 questions correctly. Find the probability that you will get at least a $B$.
c) Find the probability that you answer at most 6 questions correctly.
d) Find the probability that you answer between 3 and 6 questions correctly.
e) Find the mean and standard deviation for the number of questions that you answer correctly.
4. $6.5 \%$ of people in the U.S. have A- blood type. You randomly select 6 Americans and ask them if their blood type is A-.
a) Find the probability that all 6 are type A-.
b) Find the probability that at most 4 of them are type A-.
c) Find the mean and standard deviation.
$5.47 .5 \%$ of children say that chocolate chip cookie is their favorite kind of cookie. If you randomly select 12 children find the probability that:
a) at least 10 of them say that chocolate chip cookie is their favorite cookie.
b) four of them say that chocolate chip cookie is their favorite cookie.
c) Find the mean and standard deviation for this sample.

## Table 4-Standard Normal Distribution



| $z$ | . 09 | . 08 | . 07 | . 06 | . 05 | . 04 | . 03 | . 02 | . 01 | . 00 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.4 | .0002 | . 0003 | 0003 | 0003 | .0003 | . 0003 | . 0003 | . 0003 | 0003 | 0003 |
| -3.3 | . 0003 | .0004 | . 0004 | 0004 | . 0004 | . 0004 | . 0004 | .0005 | .0005 | . 00005 |
| -3.2 | .0005 | .0005 | ,000s | 0006 | . 0006 | . 0006 | . 0006 | . 0006 | 0007 | . 00007 |
| -3.1 | ,0007 | . 00007 | 0008 | 0008 | . 0008 | . 0008 | . 0009 | . 0009 | 0009 | . 0010 |
| -3.0 | . 00010 | .0010 | 0011 | 0011 | . 0011 | . 0012 | . 0012 | . 0013 | 0013 | . 9013 |
| -2.9 | . 0014 | 0014 | 0015 | 0015 | . 0016 | . 0016 | . 0017 | . 0018 | . 0018 | . 0019 |
| $-2.8$ | . 0019 | 0020 | 0021 | 0021 | 0022 | 0023 | .0023 | . 0024 | 0025 | 0026 |
| $-2.7$ | . 0026 | . 0027 | 0028 | . 0029 | . 0030 | . 0031 | . 0032 | . 0033 | 0034 | . 0035 |
| $-2.6$ | . 0036 | 0037 | 0038 | . 0039 | . 0040 | . 0041 | . 0043 | . 0044 | . 0045 | 0047 |
| -2.5 | . 0048 | 0049 | 0051 | . 0052 | . 0054 | . 0055 | . 0057 | . 0059 | . 0060 | 0062 |
| $-2.4$ | . 0064 | 0066 | . 0068 | . 0069 | . 0071 | 0073 | . 0075 | 0078 | , 0080 | 0082 |
| $-2.3$ | . 0084 | 0087 | 0089 | . 0091 | . 0094 | . 0096 | . 0099 | . 0102 | . 0104 | . 0107 |
| $-2.2$ | . 0110 | 0113 | . 0116 | . 0119 | . 0122 | . 0125 | . 0129 | . 0132 | . 0136 | 0139 |
| -2.1 | . 0143 | 0146 | . 0150 | . 0154 | . 0158 | 0162 | . 0166 | . 0170 | . 0174 | 0179 |
| -2.0 | . 0183 | 0188 | 0192 | 0197 | . 0202 | 0207 | . 0212 | . 0217 | . 02222 | 0228 |
| $-1.9$ | . 0233 | 0239 | 0244 | . 0250 | . 0256 | 0262 | . 0268 | . 0274 | 0281 | . 0287 |
| $-1.8$ | . 0294 | . 0301 | 0307 | . 0314 | 0322 | . 0329 | 0336 | . 0344 | . 0351 | . 0359 |
| $-1.7$ | . 0367 | 0375 | 0384 | . 0392 | . 0401 | . 0409 | . 0418 | . 0427 | . 04336 | . 0446 |
| -1.6 | . 0455 | 0465 | 0475 | . 0485 | . 0495 | . 0505 | 0516 | .0526 | 0537 | 0548 |
| $-1.5$ | . 0559 | 0571 | 0582 | . 0594 | . 06606 | . 0618 | . 0630 | . 0543 | .0655 | .0668 |
| $-1.4$ | . 0681 | . 0694 | 0708 | 0721 | 0735 | . 0749 | . 0764 | . 0778 | .0793 | 0808 |
| -1.3 | . 0823 | . 0838 | 0853 | . 0869 | .0385 | . 0901 | 0918 | . 0934 | 0951 | . 0968 |
| $-1.2$ | . 0985 | . 1003 | . 1020 | 1038 | . 1056 | . 1075 | . 1093 | . 1112 | 1131 | 1151 |
| -1.1 | . 1170 | . 1190 | . 1210 | 1230 | . 1251 | . 1271 | . 1292 | . 1314 | .1335 | 1357 |
| $-1.0$ | . 1379 | . 1401 | . 1423 | . 1446 | . 1469 | . 1492 | . 1515 | . 1539 | 1562 | 1587 |
| -0.9 | . 1611 | . 1635 | . 1660 | . 1685 | . 1711 | . 1736 | . 1762 | . 1788 | . 1814 | . 1841 |
| -0.8 | . 1867 | . 1894 | . 1922 | . 1949 | . 1977 | 2005 | . 2033 | . 2061 | .2090 | 2119 |
| -0.7 | . 2148 | 2177 | . 2206 | . 2236 | . 2266 | . 2296 | . 2327 | . 2358 | 2389 | 2420 |
| -0.6 | . 2451 | 2483 | . 2514 | . 2546 | . 2578 | 2611 | . 2643 | 2676 | 2709 | 2743 |
| -0.5 | . 2776 | 2810 | . 2843 | . 2877 | . 2912 | . 2946 | . 2981 | . 3015 | . 3050 | 3035 |
| $-0.4$ | . 3121 | . 3156 | 3192 | . 3228 | . 3264 | . 3300 | . 3336 | . 3372 | 3409 | 3446 |
| -0.3 | . 3483 | . 3520 | 35\$7 | . 3594 | . 3632 | . 3669 | . 3707 | . 3745 | . 3783 | -3821 |
| $-0.2$ | . 3859 | . 3897 | . 3936 | . 3974 | 4013 | . 4052 | . 4090 | . 4129 | . 4168 | 4207 |
| -0.1 | . 4247 | . 4286 | . 4325 | . 4364 | . 4404 | . 4443 | . 4483 | . 4522 | . 4562 | . 4602 |
| -0.0 | . 4641 | .4681 | 4721 | A761 | 4801 | . 4840 | . 4880 | . 4920 | . 4960 | 5000 |

## Critical Values

| Level of Confidence $\boldsymbol{c}$ | $\boldsymbol{z}_{\boldsymbol{c}}$ |
| :---: | :--- |
| 0.80 | 1.28 |
| 0.90 | 1.645 |
| 0.95 | 1.96 |
| 0.99 | 2575 |



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Table 4 - Standard Normal Distribution (continued)


| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | . 5000 | 5040 | . 5080 | . 5120 | . 5160 | 5199 | . 5239 | . 5279 | 5319 | . 5359 |
| 0.1 | . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 | . 5714 | . 5753 |
| 0.2 | 5793 | . 5832 | . 5871 | . 5910 | . 5948 | 5987 | . 6026 | 6064 | 6103 | 6141 |
| 20.3 | 6179 | 6217 | .625s | . 6293 | 6331 | 6368 | . 6406 | . 6443 | . 6480 | 6517 |
| 0.4 | 6554 | 6591 | 6628 | . 6664 | 6700 | 6736 | 6772 | 6808 | . 6844 | 6879 |
| 0.5 | 6915 | . 6950 | 6985 | . 7019 | . 7054 | . 7088 | .7123 | 7157 | 7190 | 7224 |
| 0.6 | 7251 | 7291 | 7324 | . 7357 | . 7389 | 7422 | . 7454 | . 7486 | . 7517 | 7549 |
| $\bigcirc 0.7$ | . 7580 | . 7611 | . 7642 | .7673 | 7704 | .7734 | . 7764 | . 7794 | 7823 | . 7852 |
| 0.8 | . 7881 | .7910 | . 7939 | 7967 | . 7995 | 8023 | 8051 | .8078 | 8106 | 8133 |
| 0.9 | 8159 | 8186 | 8212 | 8238 | 8264 | 8289 | .8315 | 8340 | . 8365 | 8369 |
| 1.0 | 8413 | 8438 | 8461 | 8485 | 8508 | . 8531 | .8554 | 8577 | 8599 | 8621 |
| 1.1 | .8643 | . 8665 | . 8686 | 8706 | 8729 | 8749 | 8770 | 8790 | 8810 | 8830 |
| 1.2 | .8849 | 8869 | .8888 | 8907 | 8925 | 8944 | 8962 | 8980 | 8997 | 9015 |
| 1.3 | .9032 | . 9049 | .9066 | . 9082 | . 9099 | . 91115 | . 9131 | -9147 | 9162 | 9177 |
| 1.4 | . 9192 | . 9207 | . 9222 | .9236 | 9251 | 9265 | . 9279 | 9292 | . 9306 | 9319 |
| 3.1.5 | . 9332 | . 9345 | . 9357 | . 9370 | .9382 | . 9394 | . 9406 | . 9418 | . 9429 | . 9441 |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | 9495 | 9505 | . 9515 | . 9525 | . 9535 | 9545 |
| , 1.7 | . 9554 | . 9564 | . 9573 | . 9582 | .9591 | 9599 | . 9608 | . 9616 | . 9625 | 9633 |
| 1.8 | .9641 | . 9649 | . 9656 | . 9664 | . 9671 | 9678 | . 9686 | .9693 | . 9699 | . 9706 |
| 迢1.9 | . 9713 | . 9719 | . 9726 | . 9732 | 9738 | . 9744 | . 9750 | . 9756 | . 9761 | . 9767 |
| 2.0 | . 9772 | . 9778 | .9783 | . 9788 | 9793 | . 9798 | . 9803 | . 9808 | 9812 | .9817 |
| 2.1 | . 9821 | . 9826 | . 9830 | . 9834 | . 9838 | . 9842 | . 9846 | . 9850 | . 9854 | . 9857 |
| 2.2 | . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | 9881 | . 9884 | . 9887 | . 9890 |
| 2.3 | . 9893 | . 9896 | . 9898 | . 9901 | . 9904 | . 9906 | . 9909 | . 9911 | . 9913 | . 9916 |
| 2.4 | . 9918 | . 9920 | . 9922 | . 9925 | . 9927 | . 9929 | 9931 | . 9932 | . 9934 | 9936 |
| 2.5 | . 9938 | . 9940 | . 9941 | . 9943 | . 9945 | . 9946 | . 9948 | . 9949 | . 9951 | . 9955 |
| 2.6 | . 9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 9961 | . 9962 | . 9963 | . 9964 |
| 32.7 | . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | .9971 | . 9972 | . 9973 | . 9974 |
| 2.8 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
| 6. 2.9 | . 99881 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9988 | . 9988 | .9986 | . 9986 |
| 3.0 | . 9987 | . 9987 | . 9987 | . 9988 | .9988 | . 9989 | . 9989 | . 9989 | . 9990 | . 9990 |
| \% 6.3 .1 | . 9990 | . 9991 | . 9991 | .9991 | . 9992 | . 9992 | . 9992 | . 9992 | .9993 | . 9993 |
| 3.2 | . 9993 | .9993 | . 9994 | .9994 | . 9994 | . 9994 | . 9994 | . 9995 | .9995 | . 9995 |
| [6) 3.3 | .9995 | . 9995 | . 9995 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | .9997 |
| 3.4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | 9998 |

## Chapter 5: Normal Probability Distributions

5.1 Introduction to Normal Distributions and the Standard Normal Distribution
a) properties of a normal distribution
b) understanding mean and standard deviation
c) graphs of normal distributions
d) the standard normal distribution and its properties
e) using the standard normal table (Appendix B)
f) finding areas under the standard normal curve and interpreting the area

### 5.2 Normal Distributions: Finding Probabilities

a) finding probabilities for normal distributions (Appendix B - Table 4) and applications to real life-situations and problems

### 5.3 Normal Distributions: Finding Values

a) finding $z$-scores when the cumulative area (probability) is given
b) finding a z -score given a percentile
c) transforming a $z$-score to an $x$-value
d) finding $x$ if the probability is given

### 5.1 Introduction to Normal Distributions and the Standard Normal Distribution

1. Find the cumulative area that corresponds to a z-score of:
a) 1.25
b) -2.31 .
2. Find the area under the standard normal curve to the right of:
a) $\mathrm{z}=1.36$
b) $\mathrm{z}=-2.67$
c) $\mathrm{z}=3.15$
3. Find the area under the standard normal curve between:
a) $\mathrm{z}=-2.55$ and $\mathrm{z}=1.08$
b) $\mathrm{z}=-1.0$ and $\mathrm{z}=1.37$
c) $\mathrm{z}=-2.4$ and $\mathrm{z}=-0.47$
d) $\mathrm{z}=0.78$ and $\mathrm{z}=2.68$

## 4. Find:

a) $\mathrm{P}(\mathrm{z}<-0.51$ or $\mathrm{z}>2.6)$
b) $\mathrm{P}(\mathrm{z}<1.4$ or $\mathrm{z}>3.04)$
c) $\mathrm{P}(-1.26<\mathrm{z}<3.25)$
d) $\mathrm{P}(0<\mathrm{z}<2.89)$

### 5.2 Normal Distributions: Finding Probabilities

1. The weights of adult male beagles are normally distributed, with a mean of 25 pounds and a standard deviation of 3 pounds. A beagle is randomly selected. Find the probability that:
a) the beagle's weight is less than 24 pounds.
b) the beagle's weight is between 23 and 26 pounds.
c) the beagle's weight is more than 29 pounds.
2. The test scores for the exam in statistics class have a mean of 78 points and standard deviation of 9 points. A student is randomly selected. Assuming the scores are normally distributed find the probability that the score is:
a) more than 95 .
b) less than 30 .
c) between 80 and 90 .
d) If there are 70 students in that class, about how many will receive between 90 and 95 points?
3. Toyota Camry: Breaking distance on a wet surface is normally distributed with a mean of 149 ft . and a standard deviation of 5.28 ft .
Find the probability that the randomly picked Toyota Camry will have a breaking distance:
a) between 140 and 149 feet,
b) more than 160 feet,
c) less than 145 feet.
d) If 90 Toyota Camries are randomly selected, about how many of those vehicles have a breaking distance less than 138 ft or more than 162 ?
4. Assume that the mean annual consumption of chocolate is normally distributed with a mean of 5.2 pounds per person and a standard deviation of 1.9 pounds per person.
a) What percent of people annually consume less than 2.7 pounds per year?
b) Would it be unusual for a person to consume about 2.7 pounds of chocolate per year?
c) In a group of 150 people, about how many consume
i) between 4.5 and 6.7 pounds of chocolate per year?
ii) more than 6 pounds?

### 5.3 Normal Distributions: Finding Values

1. Find the z -score that corresponds to a cumulative area of 0.4761 .
2. Find the $z$-score that has $12.71 \%$ of the distribution's area to its right.
3. Find the z -score for which $75 \%$ of the distribution's area lies between -z and z .
4. Find the $z$-score that corresponds to each percentile:
a) $P_{8}$
b) $P_{95}$
c) $P_{87}$
5. The monthly phone bills in a city are normally distributed with a mean of $30 \$$ and a standard deviation of $12 \$$. Find the $x$-values that correspond to $z$-scores of
a) -2.35 , b) 3.17 and c) 0.23 .

Explain the meaning of your answers.
6. Annual U.S. per capita orange use: $\mu=11.4 \mathrm{lb}, \sigma=3 \mathrm{lb}$. (note: assume normal distribution)
a) What annual per capita use of oranges represents the $10^{\text {th }}$ percentile?
b) What annual per capita use of oranges represents the third quartile ( $75^{\text {th }}$ percentile)?
c) If 275 people are randomly selected, about how many would use more than 15 pounds of oranges annually?
d) Find the minimum value that would be included in the top $15 \%$ of orange use.
e) Find the value that corresponds to the first quartile.
7. The weight of bags of pretzels are normally distributed with a mean of 150 gr . and a standard deviation of 5 gr . Bags in the upper $4.5 \%$ are too heavy and must be repackaged. Also, the bags in the lower $5 \%$ do not meet the minimum weight requirement and must be repackaged.
a) What is the range of weight for a pretzel bag that does not need to be repackaged?
b) If you randomly select 125 bags (before the weight is checked), about how many would need to be repackaged?
8. In a survey of men in the United States (ages 20-29), the height was normally distributed with a mean of 69.6 inches with a standard deviation of 3.0 inches.
a) What height represents the 47 th percentile?
b) What height represents the first quartile?
c) If 320 men are randomly selected, about how many of them are taller than 71 inches?
d) Find the minimum height in the top $22 \%$.

## Chapter 9: Correlation and Regression

### 9.1 Correlation

a) correlation coefficient (the meaning) pg. 473 (book)

### 9.2 Linear Regression

a) regression line (line of best fit); finding the equation and using the equation to calculate certain values

### 9.1 Correlation

1. Explain what it means when $r$, the correlation coefficient, is -0.87 .
2. What is the difference between $r=-0.87$ and $r=0.87$ ?
3. Explain what it means when $r$, the correlation coefficient, is 0.15 .
4. What is the difference between $r=-0.20$ and $r=0.20$ ?

### 9.2 Linear Regression

1. Age and Hours Slept: The age (in years) and the number of hours slept in a day by 10 infants are given:

| Age | 0.1 | 0.2 | 0.4 | 0.7 | 0.6 | 0.9 | 0.1 | 0.2 | 0.4 | 0.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hours | 14.9 | 14.5 | 13.9 | 14.1 | 13.9 | 13.7 | 14.3 | 13.9 | 14.0 | 14.1 |

a) Find the equation of the regression line.
b) Construct a scatter plot of the data and draw the regression line.
2. Age and Systolic blood pressure:

| Age, x | Systolic blood pressure, y |
| :---: | :--- |
| 16 | 109 |
| 25 | 122 |
| 39 | 143 |
| 45 | 132 |
| 49 | 199 |
| Do you | notice any unusual data values? |

a) Find the equation of the regression line.
b) Construct a scatter plot of the data and draw the regression line.
c) Use the regression equation to predict the systolic blood pressure for someone who is i) 30 , ii) 70 , iii) 47 , iv) 5 years old.
d) Use the regression equation to predict the age of someone who has a systolic blood pressure of i) 120 , ii) 160 .

1. A study found that the mean migration distance of the green turtle was 2200 kilometers and the standard deviation was 625 kilometers. Assume that the distances are normally distributed.
a) Find the probability that a randomly selected green turtle migrated a distance between 1900 and 2400 kilometers?
b) If you randomly select 400 green turtles and measure the migration distance for these turtles, about how many of them migrate a distance more than 2960 kilometers?
2. The probability that a person in the U.S. has O+ blood type is $38 \%$.
a) Find the probability that a randomly chosen person in the U.S. does not have type O+.
b) If four unrelated people in the U.S. are selected at random, find the probability that all four have $\mathrm{O}+$.
c) If seven unrelated people in the U.S. are selected at random, find the following probabilities:
i) least one has O+ blood type.
ii) at most 2 have $\mathrm{O}+$ blood type .
iii) exactly 6 have O+ blood type.
3. Five people are selected at random. Find the probability that all five are born on a different day of the week.
4. Data set: Amount (in \$) spent on books for a semester:

| 107 | 472 | 279 | 249 | 520 | 376 | 188 | 341 | 266 | 199 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 242 | 173 | 101 | 189 | 286 | 486 | 239 | 340 | 281 | 290 |

Construct a frequency distribution table for the data set using 5 classes.
Class Frequency Midpoint Relative frequency Cumulative Frequency

Construct a relative frequency histogram and a cumulative frequency graph (ogive).
5. Eight people need to be selected for a jury from a group of twelve men and ten women.
i) In how many different ways can you select a jury of four men and four women?
ii) Find the probability of selecting a jury of five men and three women.
iii) Find the probability of selecting a jury with at least six men.
6. One in four adults say he/she has no trouble sleeping at night. You randomly select five adults and ask if he/she has no trouble sleeping at night.
a) Find the probability that the number of people (from this group of five randomly selected adults) who say that they have no trouble sleeping is at least 4 .
b) Find the mean and standard deviation.
7. Find: $P(z<0.15$ or $z>2.30)$

## WORKSHEET-Solutions

) a) $P(1900<x<2400)=P(-0.48<z<0.32)=$
$=0.6255-0.3156=0.3099$
b) $P(x>2960)=P(z>1.22)=1-0.8888=0.1112$ $400 \times 0.1112=44.48 \approx 44$

1) a) $1-0.38=0.62$
b) $(0.38)^{4}=0.02085$
c) 1 ) $1-(0.62)^{4}=0.96478$
ii) $P(0)+P(1)+P(2)-$
$\left.={ }^{7} C_{0}(0.38) \%_{0} .62\right)^{{ }^{7}+}{ }_{7} C_{1}(0.38)^{1}(0.62)^{6}+$
$+{ }_{7} C_{2}(0.38)^{2}(0.62)^{5}=0.46411$
iii) $P(6)=7 C_{6}(0.38)^{6}(0.62)^{1}=0.01307$
(3) $\frac{7}{7} \cdot \frac{6}{7} \cdot \frac{5}{7} \cdot \frac{4}{7} \cdot \frac{3}{7}=0.1499$
(4) min $=101$
max $=520$
$\frac{\text { class width }}{}=$
$\frac{520-101}{5}=83.8 \rightarrow 84$
(5) a) ${ }_{12} C_{4} \cdot{ }_{10} C_{y}=103,950$
b) $\frac{{ }_{22} C_{5} \cdot 10 C_{3}}{{ }_{22} C_{8}}=0.29721$
c) $\frac{{ }_{12} C_{610} C_{2}+{ }_{12} C_{7}+{ }_{10} C_{1}+C_{2} C_{810} C_{0}}{{ }_{22} C_{8}}$
(6) a) $p(4)+P(5)=$
$={ }_{5}\left(4(0.25)^{4}(0.75)^{1}+{ }_{5}\left(5(0.25)^{5}(0.75)^{0}=\right.\right.$
$=0.015625$
b) $\bar{x}=n p=5 \cdot 0.25=1.25 \approx 1.3$
$s=\sqrt{n p q}=\sqrt{50.25 \cdot 0.75}=$
$=0.9682 \approx 1.0$
(7) $P(z<0.15$ or $z>2.3)=$ $=0.5596+(1-0.9893)-$ $=0.5703$


| $01-184$ | 3 | 142.5 | $3 / 20=0.15$ | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $85-2.68$ | 7 | 226.5 | $7 / 20=0.35$ | 10 |


| $69-352$ | 6 | 310.5 | 0.3 | 16 |
| :---: | :---: | :---: | :---: | :---: |
| $53-436$ | 1 | 394.5 | 0.05 | 17 |
| $37-520$ | 3 | 478.5 | 0.15 | 20 |




NUST use class bondaries

## MATH 1040

## REVIEW (EXAM I)

## Chapter 1

1. For the studies described, identify the population, sample, population parameters, and sample statistics:
a) The Gallup Organization conducted a poll of 1003 Americans in its household panel to determine what percentage of people plan to cancel their summer vacation because of the increase in gasoline prices.
b) Harris Interactive surveyed 2435 U.S. adults nationwide and asked them to rate quality of American public schools.
c) The American Institute of Education conducts an annual study of attitudes of incoming college students by surveying approximately 261,000 first-year students at 462 colleges and universities. There are approximately 1.6 million first-year college students in this country.
2. Determine whether the numerical value is a parameter or a statistics (and explain):
a) A survey of 1103 students were taken from the university with 19,500 students.
b) The 2006 team payroll of the New York Mets was $\$ 101,084,963$.
c) In a recent study of physics majors at the university, 15 students were double majoring in math.
3. Identify whether the statement describes inferential statistics or descriptive statistics:
a) Based on previous clients, a marriage counselor concludes that the majority of marriages that begin with cohabitation before marriage will result in divorce.
b) $78 \%$ of electricity used in France is derived from nuclear power.
4. Determine whether the data are qualitative or quantitative:
a) the social security numbers of the employees in the law firm
b) the zip codes of a sample of 270 customers at a local grocery store
c) the number of complaint letter received by the USPS in a given month
5. Identify the data set's level of measurement (nominal, ordinal, interval, ratio): a) numbers of touchdowns scored by a major university in five randomly selected games: 12512
b) the average daily temperatures (in degrees Fahrenheit) on seven randomly selected days
c) manuscripts rated as "acceptable" or "unacceptable"
d) the lengths (in minutes) of the top ten movies with respect to ticket sales in 2007
e) the size-class for a sample of automobiles:
subcompact compact midsize large compact large
f) the four departments of a car dealership:
sales financing parts and service customer service
g) the years of birth for students in this class
6. Decide which method of data collection you would use to collect data for the study (observational study, experiment, simulation, or survey):
a) A study where a drug is given to 57 patients and placebo to another group of 57 patients to determine if the drug has an effect on a patient's illness.
b) The ages of people living within 300 miles of your home
c) The effect of stopping the cooling process of a nuclear reactor
7. Identify the sampling technique used (random, cluster, stratified, convenience, systematic):
a) 32 sophomores, 35 juniors and 49 seniors are randomly selected from 230 sophomores, 280 juniors, 577 seniors at a certain high school.
b) To ensure customer satisfaction, every $35^{\text {th }}$ phone call received by customer service will be monitored.
c) A journalist goes to a campground to ask people how they fell about air pollution.
d) Calling randomly generated telephone numbers, a study asked 855 U.S. adults which medical conditions could be prevented by their diet.
e) A pregnancy study in Chicago, randomly selected 25 communities from the metropolitan area, then interviewed all pregnant women in these communities.
8. Would sampling technique used in 7c) lead to a biased study? Explain why.
9. A report by the California Citrus Commission stated that cholesterol levels can be lowered by drinking at least one glass of a citrus product each day.
Determine whether the survey questions is biased and why.

## SOLUTIONS:

1. a) population: all Americans; sample: 1003 Americans surveyed;
population parameter: percentage of all Americans who plan to cancel their vacation; sample statistic: percentage of those in the sample who plan to cancel their vacation b) population: all Adult Americans; 2435 selected adults population parameter: opinions of all American adults on public schools; sample statistics: opinions of those in the sample on public schools c) population: 1.6 million first-year college students; sample: 261,000 selected firstyear college students; population parameters: attitudes of all first-year college students; sample statistics: attitudes of those in the sample $(261,000$ selected $)$
2. a) statistics - only 1103 students were surveyed (not all students)
b) parameter - the entire team payroll is included
c) parameter - all physics majors were included in the study
3. a) inferential b) descriptive
4. a) qualitative b) qualitative
c) quantitative
5. a) ratio
b) interval
c) ordinal
d) ratio
e) ordinal f) nominal
g) interval
6. a) experiment
b) survey
c) simulation
7. a) stratified
b) systematic
c) convenience
d) random
e) cluster
8. Yes - most likely people who go camping do not approve of air pollution. 9. Yes - a report sponsored by the citrus industry is much more likely to reach conclusions favorable to the industry.

## Chapter 2

1. The Highway Patrol, using radar, checked the speeds (in mph) of 30 passing motorists at a checkpoint. The results are listed below:
$\begin{array}{llllllllll}44 & 38 & 41 & 50 & 36 & 36 & 43 & 42 & 49 & 48\end{array}$
$\begin{array}{llllllllll}35 & 40 & 37 & 41 & 43 & 50 & 45 & 39 & 45 & 38\end{array}$
$\begin{array}{llllllllll}50 & 41 & 47 & 36 & 35 & 40 & 42 & 43 & 48 & 33\end{array}$
Construct frequency distribution, a relative frequency distribution, and a cumulative frequency distribution using six classes. Find the midpoint of each class.
Construct a frequency histogram, a relative frequency histogram and a frequency polygon using six classes.
Construct an ogive using six classes.
If the policeman writes a ticket to anyone driving over 41 miles per hr, what percent of the cars will get a ticket?
2. Use the given frequency distribution to a) construct a cumulative frequency distribution and an ogive, b) approximate the mean of the grouped data.
Phone calls (per day)

| Class | Frequency, f |
| :--- | :---: |
| $8-11$ | 18 |
| $12-15$ | 23 |
| $16-19$ | 38 |
| $20-23$ | 47 |
| $24-27$ | 32 |

3. Listed below are the ACT scores of 40 randomly selected students at a major university:

| 18 | 22 | 13 | 15 | 24 | 24 | 20 | 19 | 19 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 16 | 25 | 14 | 19 | 21 | 23 | 25 | 18 | 18 | 13 |
| 26 | 26 | 25 | 25 | 19 | 17 | 18 | 15 | 13 | 21 |
| 19 | 19 | 14 | 24 | 20 | 21 | 23 | 22 | 19 | 17 |

Construct a frequency distribution table of the data, using eight classes.
If the university wants to accept the top $90 \%$ of the applicants, what should the minimum score be?
If the university sets the minimum score at 18 , what percent of the applicants will be accepted?
4. The data below represent the infant mortality rates (per 1,000 births) and the life expectancies for seven selected countries in Africa. Use a scatter plot to display the data.

| Infant Mortality | $\underline{63}$ | 199 | 71 | 61 | 67 | 35 | 194 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Life Expectancy | 45 | 31 | 51 | 47 | 39 | 70 | 37 |

Is there a relationship between the infant mortality and the life expectancy?
5. A study was conducted to determine how people get jobs. Four hundred subjects were randomly selected and the following are the results:

| Job Sources of <br> Survey Respondents | Frequency |
| :--- | :---: |
|  |  |
| Newspaper want ads | 69 |
| Online services | 124 |
| Executive search firms | 72 |
| Mailings | 32 |
| Networking | 103 |

Construct a pie chart and a Pareto chart of the data.
6. Use the ogive below to approximate the number in the sample


Use the ogive to approximate the number of students who said that their leisure time is at least 19.5 hrs .
7. For example 1 construct a stem-and-leaf plot and a dot plot. What can you conclude about the data? Find the mean, the median, and the mode.
8. Data set: systolic blood pressure of 17 randomly selected patients at a blood bank $\begin{array}{lllllllll}135 & 120 & 115 & 132 & 136 & 124 & 119 & 145 & 98\end{array}$
$\begin{array}{llllllll}113 & 125 & 118 & 130 & 116 & 121 & 125 & 140\end{array}$

Construct a frequency distribution and frequency histogram of the data using five classes. Approximate the mean using five classes. Find the mean, the median and mode. Are there any outliers?
9. Grade points are assigned as follows: $\mathrm{A}=4, \mathrm{~B}=3$. $\mathrm{C}=2$. $\mathrm{D}=1$, AND $\mathrm{F}=0$. Grades are weighted according to credit hours. If a student receives an A in four-unit class, a D in a two unit-class, a B in 3-unit class and a C in a three-unit class, what is the student's grade point average?
10. A student receives test scores of 78 and 82 . The student's final exam score is 88 and quiz grades are $72,81,95,84$. Each test is worth $20 \%$ of the final grade, quizzes (total) count $25 \%$ of the final grade, and the final exam is $35 \%$ of the final grade.
What is the student's mean score in class?
11. Use the data to approximate the mean heart rate of adults in the gym.

12. For the stem-and leaf plot below, find the range of the data set:

| 1 | 1 | 5 |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 6 | 6 | 6 | 7 | 8 | 9 |  |  |  |
| 2 | 7 | 7 | 7 | 8 | 8 | 9 | 9 | 9 |  |
| 3 | 0 | 1 | 1 | 2 | 3 | 4 | 4 | 5 |  |
| 3 | 6 | 6 | 6 | 7 | 8 | 8 | 9 |  |  |
| 4 | 0 | 2 |  |  |  |  |  |  |  |

13. Find the sample standard deviation:
a) $15 \quad \begin{array}{llllllll}42 & 53 & 7 & 9 & 12 & 14 & 28 & 47\end{array}$
b) $70 \quad 72 \quad 71 \quad 70 \quad 69 \quad 73 \quad 69 \quad 68 \quad 70 \quad 71$
14. In a random sample, 10 students were asked to compute the distance they travel one way to school to the nearest tenth of a mile. The data is listed below. Compute the range, variance and standard deviation of the data

$$
\begin{array}{llllllllll}
1.1 & 5.2 & 3.6 & 5.0 & 4.8 & 1.8 & 2.2 & 5.2 & 1.5 & 0.8
\end{array}
$$

15. You are the maintenance engineer for a local high school. You must purchase fluorescent light bulbs for the classrooms. Should you choose Type A with $\mu=3000$ hours and $\sigma=200$ hours, or Type B with $\mu=3000$ hours and $\sigma=250$ hours?
16. Adult IQ scores have a bell-shaped distribution with mean of 100 and a standard deviation of 15 . Use the Empirical Rule to find the percentage of adults with scores between 70 and 130 .
If 250 adults are randomly selected, about how many of them have an IQ between 85 and 130 ?
(answer: abut 204 adults)
17. The average IQ of students in a particular class is 110 , with a standard deviation of 5 . The distribution is roughly bell-shaped. Find the percentage of students with an IQ above 120.
18. Heights of adult women have a mean of 63.6 in . and a standard deviation of 2.5 in . Does Chebychev's Theorem say anything about the percentage of women with heights between 58.6 in and 68.6 in? What about the heights between 61.1 in and 66.1 in ? What about the heights between 56.1 in and 71.1 in?
If 300 women are randomly selected, using Chebychev's Theorem determine about how many are between 56.1 and 71.1 inches tall? (answer: at least 267 women)
19. Use the data given in \#2. Approximate the sample standard deviation of phone calls per day.
1) $\min =33$
SOLUTIONS (Chapter 2)
$\max =50$
$\frac{50-33}{6}=\frac{19}{6}=2.5 \rightarrow 3$





over $41 \mathrm{mph} \rightarrow$ add relative frequencies for classes: $42-44,45-47,48-50$
$0.2+0.1+0.2=0.5=50 \%$


$$
\text { b) } \begin{aligned}
& \text { use midpoints. }(9.5,135,17.5,215,25 \\
& \bar{x}=\frac{9.5 \times 18+13.523+175 \cdot 33+21.5 \times 47}{+255 \times 32} \\
& \bar{x}=\frac{2973}{158}=188
\end{aligned}
$$

(3) $\begin{aligned} \min & =12 \\ \text { max } & =26 \\ \frac{26-12}{8} & =1.75 \rightarrow 2 \\ & =2 \text { class (th }\end{aligned}$

top $90 \% \rightarrow$ all but class 12-13; minimum score $=14$

$$
29 / 40=0.725=72.5 \%
$$

t)

as the number of infont mentality decreases, the life expectancy increases ${ }^{{ }^{x}} \times$ or infant mortality ${ }^{\text {as }}$,
the life expectancy
(6) a) 80
$80-35=45$
(7) $318665798653 \quad$ You can order the data \& hove
2 stems.

| 3 | 3 |  |  |  |
| :--- | :--- | :---: | :---: | :---: |
| 3 | 556667889 |  |  |  |
| 4 | 00111223334 |  |  |  |
| 4 | 557889 |  |  |  |
| 5 | 000 |  |  |  |
| 5 |  |  |  |  |
| $11 / 30=36.7 \%$ of the cars |  |  |  |  |
| drove $40-44 \mathrm{mph}$ |  |  |  |  |

mean $=\frac{1255}{30}=41.8 \overline{3}$
median $=\frac{4+42}{2}=41.5 \mathrm{mph}$
mode -does not hove much meaning since there
are 4 values with $g=3$
(8 )-to approximate the mean using
S cases $\rightarrow$ use midporits
approximation: mean $=124.3$

- mean $=124.2$
median $=124$
mode $=125$
98 is an outher
(15) lower standard deviation $\rightarrow$ data
(9) 2.75
(10) 83.6 more consistent
(ii) 70.6
(16) $95 \%$
(12) 31
(13) $\begin{aligned} \bar{x} & =\frac{227}{9}=25.2 \\ s & =17.8\end{aligned}$
b) $\bar{x}=70.3$
(14) $\begin{aligned} & x=1.49 \\ & \text { range }=4.4, \bar{x}=3.12\end{aligned}$
(19) $s=5.1$
$s^{2}=3.3124$
$s=1.82$


## REVIEW (EXAM II)

## Section 2.5

1. The following data set lists the midterm scores received by 50 students in a chemistry class:

| 45 | 85 | 92 | 99 | 37 | 68 | 67 | 78 | 81 | 25 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 97 | 100 | 82 | 49 | 54 | 78 | 89 | 71 | 94 | 87 |
| 21 | 77 | 81 | 83 | 98 | 97 | 74 | 81 | 39 | 77 |
| 99 | 85 | 85 | 64 | 92 | 83 | 100 | 74 | 68 | 72 |
| 65 | 84 | 89 | 72 | 61 | 49 | 56 | 97 | 92 | 82 |

Find the first, second, and third quartiles of the midterms scores.
Find the interquartile range. What can you conclude from this?
Draw a box-and-whisker plot that represents this data.
2.


Interpret the given box-ad-whisker plot.
3. Find the z score for the value 88 , when the mean is 95 and the standard deviation is 5 . Would 88 be considered an unusual value?
4. The birth weights for twins are normally distributed with a mean of 2353 grams and the standard deviation of 647 grams. (assume the bell-shaped distribution)
a) Use the z -score that corresponds to each birth weight to determine which birth weight could be considered unusual: $3690 \mathrm{gr}, 1200 \mathrm{gr}, 2000 \mathrm{gr}, 2353 \mathrm{gr}$.
b) The birth weight of 2 randomly selected newborn twins are 1706 gr . and 3647 gr . Using the Empirical Rule, find the percentile that corresponds to each birth weight.
5. In a data set with a minimum value of 54.5 and a maximum value of 98.6 with 300 observations, after ordering your values you realize that 81.2 is the $187^{\text {th }}$ value. Find the percentile for 81.2.
6. For the data given in $\# 1$, find the percentile that corresponds to score 81 . What score represents the $72^{\text {nd }}$ percentile?

## SOLUTIONS:

\author{

1. $\min =21$ <br> $\max =100$ <br> Q1=67 <br> Q2=81 <br> Q3=89 <br> $\mathrm{IQR}=22$
}

Outliers: 21, 25
About $25 \%$ data between 81 and 89 .
2. $\min =154, \max =265$

Q1=180, Q2=197.5, Q3=211
About $50 \%$ of the data are between 180 and 211 .
3. $-1.4, \mathrm{NO}$
4. a) 3690 gr ( z score is 2.07 )
b) $\mathrm{z}=-1, \mathrm{z}=2$
$16^{\text {th }}$ and $97.5^{\text {th }}$ percentile $(z=-1, z=2)$
5. $62^{\text {nd }}$ percentile
6. score $81-48^{\text {th }}$ percentile $74^{\text {th }}$ percentile - score 89

## Chapter 3 - Probability

1. Assume that the probability of having a boy is 0.5 , and the probability of having a girl is 0.5 . In a family with 4 children, find the probability that:
a) all the children are girls
b) all the children are the same sex
c) there is at least 1 boy
2. In a survey of college students, 125 said that they are considering taking the next semester off, and 1,030 students said that they staying in school next semester.
a) If one college student is selected at random, find the probability that the student is considering taking the next semester off?
b) If 2 students are selected (without replacement), what is the probability that both are considering taking the next semester off?
c) If 5 students are selected (without replacement), what is the probability that at least 1 will be considering taking the semester off?
3. The distribution of blood types for 200 Americans is:

| Blood type | $\mathrm{O}+$ | $\mathrm{O}-$ | $\mathrm{A}+$ | $\mathrm{A}-$ | $\mathrm{B}+$ | $\mathrm{B}-$ | $\mathrm{AB}+$ | $\mathrm{AB}-$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number | 76 | 14 | 65 | 13 | 19 | 4 | 7 | 2 |

a) If one donor is selected at random, find the probability that he/she has the blood type A+ or A-.
b) If 2 donors are selected (w/o replacement) find the probability that both donors are type AB-.
c) If 3 donors are selected (with replacement) find the probability that at least 1 donor is type O -
4. The Heights (in inches) of all males enrolled in a section of sociology class:

6 | 5 | 5 | 6 | 6 | 6 | 8 | 9 | 9 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

7 700122234455667
If a male student is selected at random, find the probability that his height is:
a) at least 68 in
b) between 69 in and 73 in (inclusive)
c) not 65 in
5. If you roll a 6 sided die 8 times, find the probability that you will roll an odd number at least once.
6. Four people are selected at random. Find the probability:
a) all 4 are born on the same day of the week
b) all 4 are born on a different day of the week
7. Find the probability that at least 2 people in a class of 92 have the same birthday.
8. A 3-person jury must be selected at random (without replacement) from a pool of 12 people that has 6 men and 6 women. Find the probability of selecting an all-female jury.
9. You purchase 15 lottery tickets for which the probability of winning some prize on a single ticket is 1 in 12 . What is the probability that you have at least 1 winning ticket among the 15 tickets?
10. Find the probability of drawing 3 diamonds in a row from a regular deck of cards a) the drawn card is returned to the deck each time
b) the drawn card is not returned to the deck each time
11. Refer to problem \#10. Find the probabilities if you are drawing 3 fives in a row.
12. Of campus professors $60 \%$ are male, and of these, $15 \%$ work for College of Humanities. Find the following probabilities:
a) randomly selected professor is a male and works for College of Humanities.
b) randomly selected professor is a male and does not work for College of Humanities.
c) randomly selected professor is a female and works for College of Humanities.
13. At a random hospital, $55 \%$ of doctors are female. $70 \%$ of these female doctors are married. If $60 \%$ of male doctors are married, what is the probability that a randomly selected doctor is married.
14. One card is selected from a standard deck of 52 playing cards. What is the probability that the card is either a heart or a ten? What is the probability that the card is either an ace or a black card? What is the probability that the card is either a seven or a queen?
15. According to the World Factbook website, as of July 2008, 67.1\% of the U.S. population was 15-64 years old. Suppose that in a survey, 15 people were chosen at random from the population. What is the probability that all 15 were not 15-64 years old? What is the probability that at least one person is 15-64 years old?
16. A random number generator selects 6 integers from 1 to 50 . What is the probability that all 6 integers are less than or equal to 8 ?
17. Two hundred people were asked if they favor the death penalty. Of the 75 that answered "yes" to the question, 38 were male. Of the 125 that answered "no" to the question, 39 were male.
If one person is selected at random, find the probability that:
a) this person answered "yes" or was a female.
b) this person answered "no" or was a female.
c) If two people were selected at random, find the probability that both were males and both answered "yes". (without replacements)
d) If 5 people were selected at random, find the probability that at least one person answered "yes" and was a male. (without replacements)
e) If 1 person is randomly selected, given that that person is a female, what is the probability that she was against the death penalty.
18. A delivery route must include stops at 7 cities (A, B, C, D, E, F, and G). How many different routes are possible? If a route is randomly selected, find the probability that the cities are arranged in alphabetical order.
19. The representatives from the State Office is making a schedule to visit 12 universities/colleges within the state. In how many different ways can they visit 5 of these institutions within the next week?
20. How many ways can a group of five women and three men be selected from fifteen women and eleven men?
21. How many different permutations of the letters in the word PROBABILITY are there?
22. If a couple has 3 boys and 2 girls, how many gender sequences are possible? Write all the sequences.
23. How many ways can 6 people, $a, b, c, d, e$, and $f$, sit in a raw at a town meeting if $c$ and d must sit together? What if a, c and e must sit together?
24. A shipment of 15 dishwashers contains 3 defective units. The contractor has ordered 4 of these 15 units, and since each is identically packaged, the selection will be at random. In how many ways can the contractor buy 4 units, and a) receive no defective units, b) receive 1 defective unit, c) receive at least 2 non-defective unit .
d) Find the probability that the contractor will buy at least 2 defective units.
25. The Scholarship Committee has five awards for top students. They are considering 28 applicants, and 15 of the candidates are majoring in mathematics.
a) What is the probability that all five awards are given to students majoring in mathematics?
b) What is the probability that none of the five recipients are majoring in mathematics?
c) What is the probability that at least 3 recipients are majoring in mathematics?

## SOLUTIONS:

1. a) 0.0625
b) 0.125
c) 0.9375
2. a) 0.1082
b) 0.01163
c) 0.4366
3. a) 0.39
b) 0.0000503
c) 0.1966
4. a) 0.7917
b) 0.4583
c) 0.9167
5. 0.9961
6. a) 0.00292
b) 0.3499
7. $1-\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdots \frac{274}{365}$
8. 0.0909
9. 0.7289
10. a) 0.0156
b) 0.0129
11. a) 0.00046
b) 0.000181
12. a) 0.09
b) 0.51
c) insufficient info.
13. 0.655
14. a) 0.3077
b) 0.5385
c) 0.1538
15. a) $(0.329)^{15} \quad$ b) 0.99999 (almost 1)
16. 0.0000168
17. a) 0.805
b) 0.81
c) 0.0353
d) 0.6555
e) 0.6992
18. $7!=5,040, \quad \mathrm{P}=1 /(7!)=0.0002$
19. 95,040
20. ${ }_{15} C_{5} \cdot{ }_{11} C_{3}=495,495$
21. $9,979,200$
22. $\frac{5!}{3!\cdot 2!}=10 \quad 23.5!\cdot 2!=240 ; 4!\cdot 3!=144$
23. a) ${ }_{12} C_{4}=495$ b) ${ }_{12} C_{3} \cdot{ }_{3} C_{1}=660$
c) ${ }_{12} C_{2} \cdot{ }_{3} C_{2}+{ }_{12} C_{3} \cdot{ }_{3} C_{1}+{ }_{12} C_{4} \cdot{ }_{3} C_{0}=1,353$
d) $\frac{{ }_{12} C_{2} \cdot{ }_{3} C_{2}+{ }_{12} C_{1} \cdot{ }_{3} C_{3}}{{ }_{15} C_{4}}=\frac{210}{1,365}=0.1538$
24. a) $\frac{{ }_{15} C_{5}}{{ }_{28} C_{5}}=0.0306 \quad$ b) $\frac{{ }_{13} C_{5}}{{ }_{28} C_{5}}=0.0131$
c) $\frac{{ }_{15} C_{3} \cdot{ }_{13} C_{2}+{ }_{15} C_{4} \cdot{ }_{13} C_{1}+{ }_{15} C_{5} \cdot{ }_{13} C_{0}}{{ }_{28} C_{5}}=0.5722$

# REVIEW (EXAM III) <br> (sections 4.1-4.2, 5.1-5.2) 

1. The random variable $x$ represents the number of final exams that a college junior will need to take during Fall semester their junior year along with the corresponding probabilities:

| x | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{x})$ | 0.02 | 0.07 | 0.13 | 0.35 | 0.30 | 0.13 |

Verify that this is a probability distribution.
Find the mean and standard deviation for the random variable.
2. 1,500 raffle tickets are sold at $3 \$$ each for 4 prizes valued at $\$ 1,000, \$ 750, \$ 500$, and $\$ 200$. You buy one ticket. What is the expected value of your gain $/$ loss?
3. State whether the variable is discrete or continuous:
a) the blood pressures of a group of students an hour before their exam
b) the temperature in degrees Fahrenheit on December $25^{\text {th }}$ in Los Angeles
c) the number of goals scored in a soccer game
d) the number of phone calls to the math dept. on a given day
e) the height of a player on a basketball team
4. Decide whether the experiment is binomial experiment. If it is not, explain why.
a) You observe the gender of 70 kindergarteners at a local elementary school. The random variable represents the number of boys.
b) You roll a die 800 times. The random variable represents the number that appears on each roll of the die.
c) The local news station is conducting a survey asking 500 households "How many times have you used any form of public transportation within the last 2 years"?
d) Each day, Sam plays a computer game in which he has a $42 \%$ chance of winning. The random variable is the number of times Sam wins in 100 days.
5. A card is selected from a standard deck and replaced. This experiment is repeated 6 times. Identify a trial, a success, and a failure. Identify n, p, q, and x. Create a binomial probability distribution for the number of hearts that you draw (from 6 trials). Find the probability of selecting exactly 3 hearts.
6. A test consists of 20 true or false questions. To get an A, you must answer at least 18 questions correctly. If you guess on each question, what is the probability that you will get an A?
7. The probability that an individual is left-handed is 0.11 . In a class of 50 students, find the probability of finding 6 left-handed students.
8. A card is selected from a standard deck and replaced. This experiment is repeated 6 times.
a) Create a binomial probability distribution for the number of hearts that you draw (from 6 trials).
b) Find the probability of selecting: i) exactly 4 hearts, ii) at least 4 hearts.

Find the mean and standard deviation.
9. A test consists of 15 multiple-choice questions. Each questions has 4 possible answers and only one answer is correct. If you guess on each question, what is the probability that you will answer at most 4 questions correctly?
10. The probability that an individual is left-handed is 0.11 . In a class of 30 students, find the probability of finding 3 left-handed students.
Find the mean and standard deviation.
11. One in four adults is currently on a diet. In a random sample of seven adults, what is the probability that the number currently on a diet is:
a) exactly three
b) at least six
12. Use the Standard Normal Distribution Table to find the indicated probabilities:
a) $P(z<1.28)$
b) $P(z>-0.74)$
c) $P(-2.15<z<1.55)$
d) $P(z<-1.5$ or $z>1.74)$
13. IQ test scores are normally distributed with a mean of 100 and a standard deviation of 15. Find the probability that a randomly selected person has an IQ score:
a) less than 90
b) between 97 and 118
c) greater than 125 .
14. According to the U.S. National Center for Health Statistics, the distribution of cholesterol levels in teenage boys is approximately normally distributed with a mean of 170 , and a standard deviation of 30 . Levels above 200 need attention. Find the probability that a teenage boy has a cholesterol level:
a) greater than 200
b) less than 120 .
15.The lengths of pregnancies of humans are normally distributed with $\mu=268$ days and $\sigma=15$ days. A baby is premature if it is born three weeks early.
a) What percent of babies are born prematurely?
b) Out of 50 pregnancies, how many would you expect to last less than 250 days?
c) Out of 85 pregnancies, how many would you expect to last between 250 and 270 days?
16. Find the $z$-score for which $92 \%$ of the distribution's area lies between $-z$ and $z$.
17. Find the z-score for which $70 \%$ of the distribution's area lies to its right?
18. For the standard normal curve, find the z-score that corresponds to the a) $7^{\text {th }}$ percentile, b) $98^{\text {th }}$ percentile.
19. Assume that the salaries of elementary school teachers in the U.S. are normally distributed with a mean of $\$ 36,000$ and a standard deviation of $\$ 3,800$.
a) What is the cutoff salary for teachers in the top $10 \%$ ?
b) Find the salary that corresponds to the 3rd quartile?
20. A tire company finds the lifespan for one brand of its tires is normally distributed with a mean if 47,500 miles and a standard deviation of 3,000 miles. If the manufacturer is willing to replace no more than $10 \%$ of the tires, what should be the approximate number of miles for the warranty?
21. The body temperatures of adults are normally distributed with a mean of $98.6^{\circ} \mathrm{F}$ and a standard deviation of $0.44^{\circ} \mathrm{F}$.
a) Find the temperature that would correspond to $\mathrm{z}=1.75$.
b) What percentage of adults will have the temperature higher than the value that corresponds to $\mathrm{z}=1.75$ ?
22. In a survey of men in the U.S. (ages 20-29), the mean height was 69.9 inches with a standard deviation of 3.0 inches.
a) What height represents the $87^{\text {th }}$ percentile?
b) What height represents the first quartile?
c) If 350 men are randomly selected, about how many would have a height greater than 76 in?

## SOLUTIONS:

$$
\text { 1. } \begin{aligned}
0 & \leq P(x) \leq 1 \text { and } \sum P(x)=1 \\
\mu & =3.23 \approx 3.2 \\
\sigma & =1.18 \approx 1.2
\end{aligned}
$$

2. expected value $=\mu=-1.36 \$$

You can expect to lose an average
Of $\$ 1.36$ for each ticket you buy.
3. a) continuous
b) continuous
c) discrete
d) discrete
e) continuous
4. a) binomial
b) not binomial (more than 2 outcomes)
c) not binomial (more than 2 outcomes)
d) binomial
5. trial: drawing a card with replacement success: card drawn is a heart
not success: card drawn is not a heart
$\mathrm{n}=6, \mathrm{p}=0.25, \mathrm{q}=0.75, \mathrm{x}=0,1,2,3,4,5,6$

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{x})$ | 0.1780 | 0.3560 | 0.2966 | 0.1318 | 0.0330 | 0.0044 | 0.0002 |

$$
P(3)=0.1318
$$

6. 0.0002 (or $0.02 \%$ )
7. $\mathrm{P}(6)={ }_{50} C_{6}(0.11)^{6}(0.89)^{44}=0.167$
8. trial: drawing a card with replacement success: card drawn is a heart not success: card drawn is not a heart $n=6, p=0.25, q=0.75, x=0,1,2,3,4,5,6$

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{x})$ | 0.1780 | 0.3560 | 0.2966 | 0.1318 | 0.0330 | 0.0044 | 0.0002 |

$$
\begin{aligned}
& \mathrm{P}(4)=0.0330, \mathrm{P}(\text { at least } 4)=0.0376 \\
& \mu=1.5, \sigma=1.06
\end{aligned}
$$

9. 0.6865
10. a) 0.173 , b) 0.00134
11. $0.2324 \quad \mu=3.3, \sigma=1.71$
12. a) 0.2514 , b) 0.4642 , c) 0.0475
13. a) 0.1587
b) 0.0475
$\begin{array}{lllll}15 . ~ a) ~ \\ 8.08 \% & \text { b) about } 6 & \text { c) about } 37 & \text { 16. }(-1.75,1.75) & \text { 17. }-0.52\end{array}$
14. a) -1.48 b) 2.05
15. a) 99.37 degrees, b) $4.01 \%$
16. a) 0.8997 , b) 0.7704 ,
c) 0.9236 , d) 0.1077
$\begin{array}{rlr}\text { 19. a) } \$ 40,864 & \text { b }) \$ 38,546 & \text { 20. } 43,660 \\ \text { 22. a) } 73.29 \text { in } & \text { b) } 67.89 \text { in } & \text { c) about } 7\end{array}$
$\begin{array}{rlr}\text { 19. a) } \$ 40,864 & \text { b }) \$ 38,546 & \text { 20. } 43,660 \\ \text { 22. a) } 73.29 \text { in } & \text { b) } 67.89 \text { in } & \text { c) about } 7\end{array}$
$\begin{array}{rlr}\text { 19. a) } \$ 40,864 & \text { b }) \$ 38,546 & 20.43,660 \\ \text { 22. a) } 73.29 \text { in } & \text { b) } 67.89 \text { in } & \text { c) about } 7\end{array}$
