Math 6780: Not so open-ended Homework Assignment 4
Due March 29, 2007

1. Some random variables have nice properties.

   a. Show that the sum of two Poisson distributed random variables is Poisson.

   b. Show that the sum of two normally distributed random variables is normal.

   c. Do you think these are respectively the only discrete and continuous distributions with this property?

2. We tried out the empirical Bayes approach on Poisson distributed data, assuming the parameters were drawn from a gamma distribution. Assume instead that you have measured N_i successes out of n trials in a series of experiments, $i = 1 \ldots k$. We want to estimate the binomial parameter p_i for each experiment. If we assume that the p_i's are drawn from a beta distribution, things again work out nicely. The p.d.f. for a beta distribution with parameters a and b has formula

$$f(p) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} p^{a-1}(1-p)^{b-1}.$$

   A random variable with this p.d.f. has expectation $a/(a+b)$.

   a. Assume values for a and b and use a Bayesian approach to estimate p_i.

   b. Show that this estimate is a weighted average of the MLE from the data in isolation (that is, N_i successes in n trials), and the mean of the underlying beta distribution.

   c. Use all the data to come up with an estimate of a and b.