Math 5110: Homework Assignment 7
Due October 24, 2017

1. Suppose the population size of some species of organism follows the model

$$\frac{dN}{dt} = \frac{3N^2}{2+N^2} - N$$

where N is measured in hundreds.

   a. Find the equilibria.

   b. Draw the phase-line diagram.

   c. Which of the equilibria are stable, according to the stability theorem?

   d. Interpret your diagram in biological terms. Why might this population behave as it does at small values?

2. The spruce budworm model (with a type III functional response) is

$$\frac{du}{dt} = ru(1 - \frac{u}{K}) - \frac{u^2}{1+u^2}$$

after non-dimensionalizing (assume that the parameters r and K are both positive). What happens to this model if there is no density-dependence in the prey (if K is set to infinity)?

   a. Plot du/dt against u different values of r.

   b. Draw the bifurcation diagram.

   c. Explain why the results differ from those with density-dependence.

3. According to Torricelli's law of draining, the rate which a fluid flows out of a cylinder through a hole at the bottom is proportional to the square root of the depth of the water. Let y represent the depth of water in centimeters. The differential equation is

$$\frac{dy}{dt} = -c\sqrt{y}.$$

Suppose $c = 2.0\sqrt{\text{cm}}/\sec$ and that the initial condition is $y(0) = 4$.

   a. Find the solution with separation of variables and graph it. What happens at $t = 2$?

   b. How does this differ from the solution of the equation $\frac{dy}{dt} = -2y$?

   c. Check the stability with the stability theorem.