Math 5110: Homework Assignment 4
Due September 19, 2017

1. Consider a population of wombats where each adult produces r offspring, each juvenile has a probability σ of maturing into an adult, and each adult has a probability p of surviving. Suppose, however, that there is some density dependence, meaning that competition leads wombats to have lower survival or reproduction when the population is large.

   a. Write the model if per capita reproduction r is reduced by a factor $1 - A_t/K$ where A_t denotes the adult population in year t.

   b. Write the model if juvenile survival σ is reduced by a factor $1 - A_t/K$.

   c. Write the model if adult survival p is reduced by a factor $1 - A_t/K$.

   d. For (b), find the equilibrium. What are the conditions on the parameters r, σ and p for there to be a positive equilibrium? Can that equilibrium be unstable?

2. Consider a population N_t of nuthatches with immigration obeying the system of equations

$$
\begin{aligned}
N_{t+1} &= \lambda N_t + I_t \\
I_{t+1} &= h(N_t)
\end{aligned}
$$

   where $h(N)$ is a positive function and $\lambda < 1$. Suppose that the system has an equilibrium at (N^*, I^*).

   a. Find the condition on $h'(N^*)$ for the system to be unstable.

   b. What happens when the system becomes unstable if $h(N)$ is decreasing?

   c. What happens when the system becomes unstable if $h(N)$ is increasing?

3. Consider a lung obeying the following:

$$
\begin{aligned}
c_{t+1} &= (1 - q_t)(c_t + A) + q_t \gamma \\
q_{t+1} &= \alpha q_t + (1 - \alpha) h(c_t).
\end{aligned}
$$

   Here, c_t is the concentration of some chemical (perhaps carbon dioxide), q_t is the fraction of air exchanged during breath t, A is the amount of chemical produced by the lung, and γ is the external concentration. The body responds to a high concentration by increasing the fraction exchanged according to the function $h(c_t)$, but cannot adjust immediately.

   a. First, suppose that q_t is constant. Find the equilibrium of the equation for c_t in this case. Show that the equilibrium is stable as long as $q_t > 0$.

   b. Now set $A = 1/3$, $\alpha = 1/2$, $\gamma = 2/3$ and

$$
h(c) = \frac{c^n}{1 + c^n}.
$$

   Check that $q^* = 1/2$ and $c^* = 1$ is an equilibrium.

   c. Option 1 Find the Jacobian matrix for this system. Is it possible for the equilibrium to become unstable? If so, what kind of bifurcation will it undergo, and what are the resulting dynamics?

   c. Option 2 Modify the R code medcontrol.R (or write your own program) to investigate whether the equilibrium can become unstable. If it does, what kind of bifurcation did it undergo?