

1. Consider a population consisting of juveniles and adults, where each adult produces m offspring, each juvenile has a probability σ of maturing into an adult, and each adult has a probability p of surviving. In addition, however, a fraction γ of the juveniles follow the “Peter Pan” strategy and remain alive as juveniles (a fraction $1 - \sigma - \gamma$ die).

- (a) Write the matrix describing this situation.
- (b) Suppose $m = 3.75$ and $p = 0.5$ and that $\sigma + \gamma = 0.5$. Find the eigenvalues when $\sigma = 0.5$.
- (c) Find the eigenvalues when $\sigma = 0.25$.
- (d) Find the eigenvalues when $\sigma = 0.0$. Could you have guessed the leading eigenvalue without doing any calculations?
- (e) Which population will grow the most quickly? How do the stable age distributions compare (which population will be the most juvenile)?

2. Suppose a population can experience two types of years. In the first, $m = 0.5$, $p = 0.5$ and $\sigma = 0.9$ (with m , p and σ defined as in the previous problem). In the second, $m = 2.0$, $p = 0.5$ and $\sigma = 0.2$.
- (a) Find the leading eigenvalue for each population. What would happen to each in the long run?
 - (b) Suppose instead that the two types of years **alternate**. Find the matrix describing what the population looks like after two years.
 - (c) Find the leading eigenvalue of this two year matrix. Will the population grow or shrink?
 - (d) Explain how this apparently baffling paradox is possible.

3. Suppose individuals of age 0 have a probability p of surviving to age 1, individuals of age 1 have a probability p of surviving to age 2, and so on for all ages. Furthermore, individuals of all ages (including age 0) produce m offspring per year.

- (a) Write a Leslie matrix describing this population.
- (b) Summarize your Leslie matrix as a 2×2 matrix.
- (c) Find the leading eigenvalue of this matrix.
- (d) Think of m as being large, say, 1000. Suppose a mutant arises that produces one more offspring at age 0, but dies from the effort. What is the long term rate of growth of a population of mutants? Which grows faster, the population of mutants or the original population?
- (e) This result is known as Cole's Law, after Lamont Cole, a famous ecologist and connoisseur of picnic salads. Does this result surprise you? Why would any organism bother surviving more than one year?