Math 5110: Homework Assignment 15
Due on December 12, 2017

1. Let's use linearization to analyze a version of the Fitzhugh-Nagumo model, and to see the dynamics that can result when a constant current is applied.

$$\begin{aligned} \frac{dv}{dt} &= f(v) - w \\ \frac{dw}{dt} &= \epsilon(w_\infty(v) - w). \end{aligned}$$

The function $f(v)$ has a cubic shape such that

$$\begin{aligned} f(0) &= 0,\ f(a) = 0,\ f(1) = 0 \\ f'(0) &< 0,\ f'(a) > 0,\ f'(1) < 0 \end{aligned}$$

for $0 < a < 1$. The function $w_\infty(v)$ is increasing with $w_\infty(0) = 0$. The parameter ϵ is small.

a. Find the Jacobian matrix and show that the equilibrium at $v = w = 0$ must be stable.

b. A constant current I_a can be added to the right hand side of the differential equation for v. Draw a phase plane for this situation with a single equilibrium.

c. Using the Jacobian, find the conditions under which this equilibrium is unstable.

d. Sketch a solution in the phase plane in this case and use it to graph the behavior of v as a function of time (keeping in mind that ϵ is small. What good might this be?