Math 5110: Homework Assignment 11
Due on November 21, 2017

1. Consider again the basic reaction
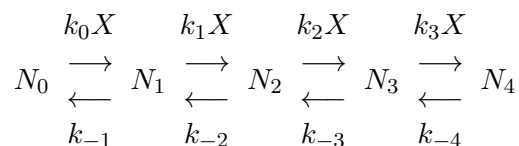
$$E + S \; \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} \; C \; \overset{k_2}{\longrightarrow} \; E + P$$
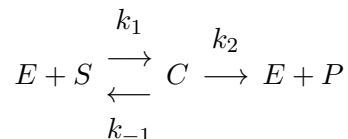
   with the initial conditions $S(0) = S_0$, $E(0) = E_0$ and $C(0) = 0$. In class, we scaled C by E_0. Instead, scale C by its value on the C-nullcline at $S = S_0$. What is the small parameter?

2. Consider the reaction

$$A \; \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} \; B \; \overset{k_2}{\longrightarrow} \; C$$
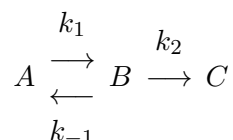
   with the initial conditions $A(0) = A_0$ and $B(0) = C(0) = 0$.

   a. Write the differential equations.

   b. Scale A by A_0. If A were stuck at A_0, what would the value of B approach? Use this value to scale B.

   c. Suppose k_1 is large relative to the other rates. Find the inner and outer solutions of the equations (the "zip" and "putt-putt" parts).

3. The following describes a molecule, such as hemoglobin, which can bind four smaller molecules, such as oxygen. The oxygen concentration is given by X, and the fraction of hemoglobin molecules with i oxygens is N_i.

$$N_0 \; \underset{k_{-1}}{\overset{k_0 X}{\rightleftharpoons}} \; N_1 \; \underset{k_{-2}}{\overset{k_1 X}{\rightleftharpoons}} \; N_2 \; \underset{k_{-3}}{\overset{k_2 X}{\rightleftharpoons}} \; N_3 \; \underset{k_{-4}}{\overset{k_3 X}{\rightleftharpoons}} \; N_4$$

   Find the equilibrium fraction in state N_4 as a function of X. Under what conditions does this reduce to a Michaelis-Menten form?