Math 5110: Homework Assignment 10
Due November 14, 2017

1. The Poisson distribution can be thought of as the result of an immigration process. A population of size i increases to $i+1$ at rate λ (not per capita).

   a. How does this differ from a birth process where each individual gives birth at rate λ?

   b. Write the system of differential equations describing this process.

   c. Find the mean as a function of time.

   d. Find the formula giving the probability of i individuals at time t.

2. Suppose each individual in a population gives birth to twins (thus increasing the population by 2) at rate λ and dies at rate μ. Find the probability of extinction starting from a population of size i. How does it compare with the result when individuals give birth to a single offspring?

3. Consider a population of fixed size K distributed on two islands. Each individual on island 1 migrates to island 2 independently at probabilistic rate μ_1 while each individual on island 2 migrates to island 1 independently at probabilistic rate μ_2.

   a. Draw a diagram illustrating the process.

   b. Write the differential equations for the probabilities.

   c. Suppose $K = 2$. Find the equilibrium probabilities for p_0, p_1 and p_2. Take a guess at the equilibrium probabilities for general K.