Math 5110: Homework Assignment 1
Due August 31, 2017

1. A population of bacteria has been engineered to produce a valuable medication, but must be harvested to collect it. Suppose the bacteria follow

$$b_{t+1} = rb_t - h.$$

The bacteria have per capita production $r > 1$, and h are harvested each generation.

a. Find the equilibrium population. Are you worried that the equilibrium is an increasing function of h?

b. Find the solution. What happens if the population starts out below the equilibrium?

c. Can you think of a more intelligent way to choose the number of bacteria to harvest each generation?

2. A population of myrtle spurge, a noxious weed, is spreading in the foothills of the Salt Lake Valley. Suppose that plants produce $\lambda < 1$ offspring that sprout the next year and then die. People continue to plant these in their gardens, and each year, I of them escape to join the wild population. Faced by this problem, officials propose two control policies:

- kill a fraction c of the adult plants each summer before they reproduce, reducing λ by a factor of $1 - c$,
- reduce the number that escape into the wild to kI where $k < 1$.

Our goal is to see how effective these policies are alone or in combination. If you want to, set $I = 100$ and $\lambda = 0.99$.

a. Find the equilibrium number S^* of wild myrtle spurge if $c = 0$ and $k = 1$ (the situation without control measures).

b. Graph S^* as a function of k when $c = 0$ and when $c = 1$. Explain your graphs; why are they increasing or decreasing, and why is one higher than the other?

c. Graph S^* as a function of c when $k = 1$ and when $k = 0.1$. Explain these graphs as before.

d. What do you think makes the best control measure for these plants? For example, if it costs as much to set $k = 0.5$ or $c = 0.5$, which strategy is more effective? Why?