Math 5110: Homework Assignment 9
Due on November 15, 2005

1. Consider again the basic reaction

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} C \overset{k_2}{\longrightarrow} E + P$$

with the initial conditions $S(0) = S_0$, $E(0) = E_0$ and $C(0) = 0$. In class, we scaled C by E_0. Instead, scale C by its value on the C-nullcline at $S = S_0$. What is the small parameter?

2. Consider the reaction

$$A \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} B \overset{k_2}{\longrightarrow} C$$

with the initial conditions $A(0) = A_0$ and $B(0) = C(0) = 0$.

a. Write the differential equations.

b. Scale A by A_0. If A were stuck at A_0, what would the value of B approach? Use this value to scale B.

c. Suppose k_1 is large relative to the other rates. Find the inner and outer solutions of the equations (the "zip" and "putt-putt" parts).

d. Suppose k_1 is small relative to the other rates. Find the inner and outer solutions of the equations. How does this differ from the usual analysis?

3. Consider the autocatalytic reaction

$$A + X \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} 2X$$

Analyze the model with the following two assumptions.

a. Suppose that A is depleted rather than held constant as in class. Find the equilibrium and solution. Does this make any difference in the short term?

b. Suppose that A is supplemented at constant rate S. How fast is X produced in the long run?

4. Suppose a receptor makes X when in the unbound state U, but is bound by X and represses the production of X according to

$$X + U \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} B$$

$$U \overset{k_2}{\longrightarrow} X \overset{\delta}{\longrightarrow} \text{trash}$$

Suppose that $U+B=1$ (so that U and B are the fractions of receptors in each state), and that $U(0)=1$, $X(0)=B(0)=0$.

a. Find the solution if $k_1=k_{-1}=0$.

b. Does tying up the receptors in the bound state B slow down the reaction? That is, what happens when $k_1 \neq 0$ and $k_{-1} \neq 0$?

5. THIS PROBLEM IS NOT REQUIRED, AND IS NOT EXTRA CREDIT. IT'S JUST FOR THOSE WHO WANT AN EXTRA PROBLEM TO DO.
The following describes a molecule, such as hemoglobin, which can bind four smaller molecules, such as oxygen. The oxygen concentration is given by X, and the fraction of hemoglobin molecules with i oxygens is N_i.

$$N_0 \underset{k_{-1}}{\overset{k_0 X}{\rightleftarrows}} N_1 \underset{k_{-2}}{\overset{k_1 X}{\rightleftarrows}} N_2 \underset{k_{-3}}{\overset{k_2 X}{\rightleftarrows}} N_3 \underset{k_{-4}}{\overset{k_3 X}{\rightleftarrows}} N_4$$

Find the equilibrium fraction in state N_4 as a function of X. Under what conditions does this reduce to a Michaelis-Menten form?