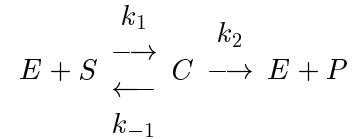


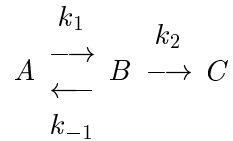
**Math 5110: Homework Assignment 9**  
**Due on November 15, 2005**

1. Consider again the basic reaction



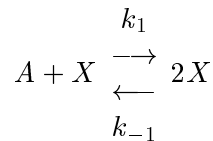
with the initial conditions  $S(0) = S_0$ ,  $E(0) = E_0$  and  $C(0) = 0$ . In class, we scaled  $C$  by  $E_0$ . Instead, scale  $C$  by its value on the  $C$ -nullcline at  $S = S_0$ . What is the small parameter?

2. Consider the reaction



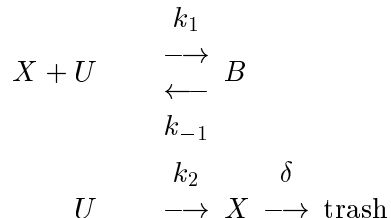
with the initial conditions  $A(0) = A_0$  and  $B(0) = C(0) = 0$ .

- a. Write the differential equations.
  - b. Scale  $A$  by  $A_0$ . If  $A$  were stuck at  $A_0$ , what would the value of  $B$  approach? Use this value to scale  $B$ .
  - c. Suppose  $k_1$  is large relative to the other rates. Find the inner and outer solutions of the equations (the “zip” and “putt-putt” parts).
  - d. Suppose  $k_1$  is small relative to the other rates. Find the inner and outer solutions of the equations. How does this differ from the usual analysis?
3. Consider the autocatalytic reaction



Analyze the model with the following two assumptions.

- a. Suppose that  $A$  is depleted rather than held constant as in class. Find the equilibrium and solution. Does this make any difference in the short term?
  - b. Suppose that  $A$  is supplemented at constant rate  $S$ . How fast is  $X$  produced in the long run?
4. Suppose a receptor makes  $X$  when in the unbound state  $U$ , but is bound by  $X$  and represses the production of  $X$  according to



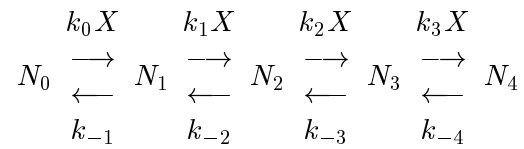
Suppose that  $U + B = 1$  (so that  $U$  and  $B$  are the fractions of receptors in each state), and that  $U(0) = 1, X(0) = B(0) = 0$ .

a. Find the solution if  $k_1 = k_{-1} = 0$ .

b. Does tying up the receptors in the bound state  $B$  slow down the reaction? That is, what happens when  $k_1 \neq 0$  and  $k_{-1} \neq 0$ ?

5. THIS PROBLEM IS NOT REQUIRED, AND IS NOT EXTRA CREDIT. IT'S JUST FOR THOSE WHO WANT AN EXTRA PROBLEM TO DO.

The following describes a molecule, such as hemoglobin, which can bind four smaller molecules, such as oxygen. The oxygen concentration is given by  $X$ , and the fraction of hemoglobin molecules with  $i$  oxygens is  $N_i$ .



Find the equilibrium fraction in state  $N_4$  as a function of  $X$ . Under what conditions does this reduce to a Michaelis-Menten form?