Math 5110: Homework Assignment 6
Due October 25, 2005

1. We have seen that the disease model

$$\frac{dS}{dt} = \beta S - \alpha IS$$
$$\frac{dI}{dt} = \alpha IS - \mu I$$

is equivalent to the Lotka-Volterra predator-prey model.

   a. Suppose first that diseased individuals recover at rate γ. Analyze everything (i.e. find the equilibria and their stability and draw a nice phase-plane). Does this make sense as a predator-prey equation?

   b. Suppose instead that diseased individuals reproduce and produce uninfected offspring, but do so at a reduced rate (multiply β by a value less than 1). Analyze everything as before.

2. It is a sad fact that cannibalism is rampant in the natural world. One model describing this is

$$\frac{dJ}{dt} = r(J)A - \mu AJ - \theta J$$
$$\frac{dA}{dt} = \theta J - \delta A.$$

Suppose that $r(J)$ is a linearly increasing function $r(J) = r_0 + \eta\mu J$ with $\eta < 1$.

   a. Explain every equation.

   b. Draw the phase-plane in all its glory.

   c. Find every equilibrium and its stability.

   d. Under what conditions can this population survive?

3. The model of predator functional response describes how the predator attack rate might depend on the prey population size. Alternatively, the prey might choose a behavior that reduces the attack rate at some cost (such as a reduced birth rate).

$$\frac{dN}{dt} = r(P)N - \eta(P)NP$$
$$\frac{dP}{dt} = a\eta(P)NP - \delta P.$$

Suppose that $r(P)$ is the linearly decreasing function $r(P) = r_0 - r_1 P$ $\eta(P)$ is the linearly decreasing function $\eta(P) = \eta_0 - \eta_1 P$.

   a. Explain every equation.

   b. Find every equilibrium and its stability. What are the conditions for existence of an equilibrium?

   c. Draw the phase-plane.