## Math 5110: Homework Assignment 5
## Due October 18, 2005

1. The logistic equation is

$$\frac{dN}{dt} = rN(1 - \frac{N}{K}).$$

Solve this equation two different ways: (1) separating variables and integrating and (2) treating it as a Bernoulli equation (this means that you use a substitution of the form $v = N^m$ for some power $m$ that transforms the original equation into a linear equation).

2. Suppose the population size of some species of organism follows the model

$$\frac{dN}{dt} = \frac{3N^2}{2 + N^2} - N$$

where $N$ is measured in hundreds.

   **a.** Find the equilibria.

   **b.** Draw the phase-line diagram.

   **c.** Which of the equilibria are stable, according to the stability theorem?

   **d.** Interpret your diagram in biological terms. Why might this population behave as it does at small values?

3. The spruce budworm model (with a type III functional response) is

$$\frac{du}{dt} = ru(1 - \frac{u}{K}) - \frac{u^2}{1 + u^2}$$

after non-dimensionalizing (assume that the parameters $r$ and $K$ are both positive). What happens to this model if there is no density-dependence in the prey (if $K$ is set to infinity)? Find the different behaviors possible for the model, and try to explain why the results are different from those with density-dependence.

4. According to Torricelli's law of draining, the rate which a fluid flows out of a cylinder through a hole at the bottom is proportional to the square root of the depth of the water. Let $y$ represent the depth of water in centimeters. The differential equation is

$$\frac{dy}{dt} = -c\sqrt{y}.$$

Suppose $c = 2.0\sqrt{\text{cm}}/\sec$ and that the initial condition is $y(0) = 4$.

   (a) Find the solution with separation of variables and graph it. What happens at $t = 2$?

   (b) How does this differ from the solution of the equation $\frac{dy}{dt} = -2y$?

   (c) Check the stability with the stability theorem.