

Math 5110: Homework Assignment 4
Due September 27, 2005

1. Consider the Nicholson-Bailey model in the case that each parasitoid produces c offspring rather than just one as in the standard equations. Find the equilibria and their stability. Can low parasitoid reproduction stabilize the equations?
2. Suppose that a population of hosts and parasitoids follow the Nicholson-Bailey equations but that each year a fraction p of the hosts have a safe refuge from attack. Find the equilibria and their stability. Is this different from a case where there are S safe sites, and only $N_t - S$ hosts are available for attack?
3. Suppose that instead of having a safe place to hide, hosts have a safe age. In particular, adults are vulnerable to attack, with a proportion e^{-aP_t} surviving long enough to make λ offspring. However, offspring are not attacked by parasitoids, and a fraction of σ survive to adulthood. Development takes exactly two years for hosts and one year for the parasitoids. Under what conditions will this invulnerable age class stabilize the interaction?
4. Suppose that the distribution of vulnerability to attack follows a gamma distribution with mean 1, given by the probability density function

$$f(\theta) = k^k \theta^{k-1} e^{-k\theta} / \Gamma(k)$$

where $\Gamma(k)$ is the Γ -function, needed to make sure that $f(\theta)$ is in fact a probability density function. The probability that a host with vulnerability θ survives attack is $e^{-a\theta P}$.

- a. Find the probability that a host survives as a function of P .
- b. What values of k stabilize the equilibrium of the host-parasitoid equations?
- c. What do small and large values of k mean biologically? Think about both the shape of $f(\theta)$ and the answer to part a.

It might help to use the following facts about the Γ -function:

$$\begin{aligned}\Gamma(z) &= \int_0^\infty t^{z-1} e^{-t} dt \\ \Gamma(z+1) &= z\Gamma(z).\end{aligned}$$