Math 5110: Homework Assignment 3
Due September 20, 2005

1. Consider the population where each adult produces r offspring, each juvenile has a probability σ of maturing into an adult, and each adult has a probability p of surviving. Suppose, however, that there is some density dependence.

   a. Find the equilibrium population if per capita reproduction r is reduced by a factor $1 - a/K$ when the adult population is a.

   b. Find the equilibrium population if juvenile survival σ is reduced by a factor $1 - a/K$ when the adult population is a.

   c. Find the equilibrium population if adult survival p is reduced by a factor $1 - a/K$ when the adult population is a.

   d. For each case, find the condition on the parameters r, σ and p needed to make this equilibrium positive. Can any of these equilibria be unstable?

2. Consider a population of size N_t with immigration obeying the system of equations

$$\begin{aligned} N_{t+1} &= \lambda N_t + I_t \\ I_{t+1} &= h(N_t) \end{aligned}$$

   where $h(N)$ is a positive decreasing function and $\lambda < 1$. Suppose that the system has an equilibrium at (N^*, I^*)

   a. Find the condition on $h'(N^*)$ for the system to be unstable.

   b. Suppose that $h(N) = e^{-\alpha N}$. Show that no values of λ and α can lead to unstable dynamics.

3. Consider a lung obeying the following

$$\begin{aligned} c_{t+1} &= (1 - q_t)(c_t + A) + q_t \gamma \\ q_{t+1} &= \alpha q_t + (1 - \alpha) h(c_t). \end{aligned}$$

   Here c_t is the concentration of some chemical (perhaps carbon dioxide) q_t is the fraction of air exchanged during breath t, A is the amount of chemical produced by the lung, and γ is the external concentration. The body responds to a high concentration by increasing the fraction exchanged according to the function $h(c_t)$, but cannot adjust immediately.

   a. First, suppose that q_t is constant. Find the equilibrium of the equation for c_t in this case. Show that the equilibrium is stable as long as $q_t > 0$.

   b. Now set $A = 1/3$, $\alpha = 1/2$, $\gamma = 2/3$ and

$$h(c) = \frac{c^n}{1 + c^n}.$$

   Check that $q^* = 1/2$ and $c^* = 1$ is an equilibrium.

   c. Option 1 Find the Jacobian matrix for this system. Is it possible for the equilibrium to become unstable? If so, what kind of bifurcation will it undergo, and what are the resulting dynamics?

   c. Option 2 Use the matlab code posted on the web site to investigate whether the equilibrium can become unstable. If it does, what kind of bifurcation did it undergo?