Math 5110: Homework Assignment 10
Due on November 22, 2005

1. Find the Jacobian of the Hodgkin-Huxley "fast" plane,

$$\begin{aligned} C\frac{dV}{dt} &= \bar{g}_{Na}m^3h(0)(V_{Na}-V)+\bar{g}_K n(0)^4(V_K-V)+\bar{g}_L(V_L-V) \\ 0.4\frac{dm}{dt} &= m_\infty(v)-m \end{aligned}$$

assuming that m_∞ is an increasing function and that all parameters are positive except V_K. Show that this system cannot have an equilibrium with complex eigenvalues. What is the condition for an equilibrium to be stable?

2. A highly simplified version of the Fitzhugh-Nagumo equations assumes that $f(v) = -v(v-a)(v-1)$ and that $w_\infty(v) = v/\gamma$ for some parameter γ. Consider this system with positive applied current I_a.

$$\begin{aligned} \frac{dv}{dt} &= -v(v-a)(v-1)-w+I_a \\ \frac{dw}{dt} &= \epsilon(v-\gamma w). \end{aligned}$$

   a. Find necessary conditions on the parameters to create an equilibrium that has complex eigenvalues with positive real parts. Draw the phase-plane in this case. Sketch a solution in the phase-plane and the resulting trajectory of v as a function of time.

   b. Is it possible for the w-nullcline to intersect the increasing portion of the v-nullcline without producing an oscillation? If so, what does the system do?

3. The Morris-Lecar model provides an alternative reduction of Hodgkin-Huxley dynamics to two dimensions, and follows calcium (Ca) in place of sodium. A simplified version is

$$\begin{aligned} \frac{dV}{dt} &= \bar{g}_{Ca}M_\infty(V)(V_{Ca}-V)+\bar{g_K}W(V_K-V) \\ \frac{dW}{dt} &= \phi(W_\infty(V)-W). \end{aligned}$$

where

$$\begin{aligned} M_\infty(V) &= \frac{1}{2}(1+\tanh(\frac{V}{15}) \\ W_\infty(V) &= \frac{1}{2}(1+\tanh(\frac{V}{30}). \end{aligned}$$

The other constants are $V_{ca} = 100$, $V_K = -70$, $\bar{g}_{Ca} = 4.4$ and $\bar{g}_K = 8$, and $\phi = 0.1$.

   a. Sketch graphs of $M_\infty(V)$ and $W_\infty(V)$. Which functions do these correspond to in the Hodgkin-Huxley set-up?

   b. Using these parameter values, use a computer or graphics calculator to draw the phase-plane. How do solutions behave?

   c. FOR EXTRA WORK: Why are the parameters that scale $M_\infty(V)$ and $W_\infty(V)$ different? What happens if the 15 and 30 are switched? Can you explain this biologically?