Math 5110: Homework Assignment 1
Due September 6, 2005

1. Consider a population consisting of juveniles and adults, where each adult produces m offspring, each juvenile has a probability σ of maturing into an adult, and each adult has a probability p of surviving. In addition, however, a fraction γ of the juveniles follow the "Peter Pan" strategy and remain alive as juveniles (a fraction $1 - \sigma - \gamma$ die).

   a. Write the matrix describing this situation.

   b. Suppose $m = 3.75$ and $p = 0.5$ and that $\sigma + \gamma = 0.5$. Find the eigenvalues when $\sigma = 0.5$.

   c. Find the eigenvalues when $\sigma = 0.25$.

   d. Find the eigenvalues when $\sigma = 0.0$. Could you have guessed the leading eigenvalue without doing any calculations?

   e. Which population will grow the most quickly? How do the stable age distributions compare (which population will be the most juvenile)?

2. Suppose a population can experience two types of years. In the first, $m = 0.5$, $p = 0.5$ and $\sigma = 0.9$ (with m, p and σ defined as in the previous problem). In the second, $m = 2.0$, $p = 0.5$ and $\sigma = 0.2$.

   a. Find the leading eigenvalue for each population. What would happen to each in the long run?

   b. Suppose instead that the two types of years **alternate**. Find the matrix describing what the population looks like after two years.

   c. Find the leading eigenvalue of this two year matrix. Will the population grow or shrink?

   d. Explain how this apparently baffling paradox is possible.

3. Suppose individuals of age 0 have a probability p of surviving to age 1, individuals of age 1 have a probability p of surviving to age 2, and so on for all ages. Furthermore, individuals of all ages (including age 0) produce m offspring per year.

   a. Write a Leslie matrix describing this population.

   b. Summarize your Leslie matrix as a 2×2 matrix.

   c. Find the leading eigenvalue of this matrix.

   d. Think of m as being large, say 1000. Suppose a mutant arises that produces one more offspring at age 0, but dies from the effort. What is the long term rate of growth of a population of mutants? Which grows faster, the population of mutants or the original population?

   e. This result is known as Cole's Law, after Lamont Cole, a famous ecologist and connoisseur of picnic salads. Does this result surprise you? Why would any organism bother surviving more than one year?

4. The following is based on problem 16 in chapter 1 of the Edelstein-Keshet book. We track the number of red blood cells (RBCs), which should remain roughly constant over time. Let

$$\begin{aligned} R_t &= \text{number of RBCs in circulation on day } t \\ M_t &= \text{number of RBCs produced by bone marrow on day } t \\ f &= \text{fraction of RBCs removed each day} \\ \gamma &= \text{a production constant} \end{aligned}$$

and suppose they follow the equations

$$\begin{aligned} R_{t+1} &= (1-f)R_t + M_t \\ M_{t+1} &= \gamma f R_t. \end{aligned}$$

a. Try to make sense of these equations, with particular attention to meaning of the parameter γ and the distinction between R_t and M_t. Are blood cells ready to get to work right after being produced?

b. Find the eigenvalues and determine their signs and magnitudes.

c. Show that numbers of RBCs will remain constant if the larger eigenvalue is equal to 1. What is the value of γ?

d. In this case, what is the other eigenvalue? Describe the resulting dynamics.