

**Math and Medicine: Homework Assignment 8**  
**Due on October 27**

1. Lloyd's mean crowding states, rather paradoxically, that most people live in places that are more crowded than average.
  - a. Suppose people live in cities of sizes 1,000,000 and 10,000,000. If there are an equal number of cities of these two sizes, what fraction of people live in places more crowded than average?
  - b. What is the mean city size from the perspective of a randomly chosen resident? Show that this has the same formula as for length-biased sampling.
2. Physicists love power laws, which means that the distribution of something declines not exponentially, but like a power. For example, the duration of time spent in the hospital might be

$$f(\tau) = \frac{p}{\tau^{p+1}}$$

for  $\tau \geq 1$  and  $p \geq 0$ .

- a. Suppose that  $p = 3$ . Check that  $\int_1^\infty f(\tau) d\tau = 1$ . Find the mean and variance of times in the hospital, and the extent of length biased sampling.
- b. Try the same steps with  $p = 2$ . What goes wrong? If you can figure out how to simulate this process, does the mathematical disaster seem to make any sense?
- c. If you haven't gone mad, try the same with  $p = 1$ .