Math and Medicine: Homework Assignment 7
Due on October 20

1. Consider a multistage carcinoma model where the mutation rate of a tissue from stage i (with i "hits") to stage $i+1$ is $(i+1)\mu$ (that is, the mutation rate from stage 1 to stage 2 is double that of stage 0 to stage 1, that from stage 2 to stage 3 is triple that of stage 0 to stage 1, and so forth).

   a. Write the equations for a k-hit cancer.

   b. Find how incidence $I(a)$ depends on age for a two-hit cancer. Does the graph of $\ln(I(a))$ against $\ln(a)$ fall approximately along a line? If so, what is the slope of that line?

   c. **Extra Credit**: How well does this work for a k-hit cancer?

2. Some cancer-associated viruses, like hepatitis C, do not directly cause cancer, but just increase cell turnover rates. Suppose infection increases the cell turnover rate by a factor of θ.

   a. Write differential equations for the populations of cells S_i with i hits, assuming that all cells begin in stage 0, and that the mutation rate from stage i to $i+1$ is μ.

   b. Is the incidence of cancer increased by a factor of θ for a two-hit cancer?

   c. Is the incidence increased by a factor of θ or θ^{k-1} for a k-hit cancer?