Math and Medicine: Homework Assignment 5
Due on September 29

Problems 1 and 2 are extensions of the calculations done in class. Pick the one that is more interesting and work out all the details. If they are exactly equally interesting, do both! Problem 3 is just drawing pictures, so do that one also.

1. Our model ignores maternal antibodies, by which babies born to resistant mothers enter an M category, but remain immune for only about one year.

   a. Sketch a diagram of this case. Assume that people who are born into the M class leave at rate ρ to enter the S class.

   b. Write a system of four differential equations for S, M, I and R.

   c. Follow the steps from class to find the equilibrium, using the same parameter values (death rate of 1/70 per year, recovery rate of 40/year, and $R_0 = 6$). How much difference do maternal antibodies make for the equilibrium and the mean age of infection?

   d. **Extra credit:** Modify the program on the website to include a class with maternal antibodies.

2. Our model assumes people are vaccinated at birth and thus are born directly into the R category. Suppose instead that people are vaccinated at rate ν starting at birth, and enter the R class.

   a. Sketch a diagram of this case.

   b. Write a system of three differential equations for S, I, and R.

   c. Follow the steps from class to find the equilibrium, using the same parameter values (death rate of 1/70 per year, recovery rate of 40/year, and $R_0 = 6$), and experiment with ν to find the equilibrium and the mean age of infection.

   d. Is there a best ν to minimize CRS?

   e. **Extra credit:** Modify the program on the website to include this.

3. Sketch a diagram and word equations for the following cases.

   a. Vaccination that occurs at some fixed age (rather than a rate).

   b. A vaccination that loses effectiveness.

   c. One other model extension that would be interesting.