NAME:_____

MATH 1180
Fake Midterm II

Do all **four** problems. One page of notes, no calculators, no cheating.

1. A recent study of viral infections looked at three ages of participants: preschoolers (age 2, 40% of people), school-aged children (age 10, 40% of people), and adults (age 25, 20% of people). On average, 50% of the preschoolers, 40% of the school-aged children, and 20% of the adults are infected.

a. Define two random variables, one for age and one for infection. How many different values does each take on?

b. Write the given information in terms of your random variables.

c. Which information describes a marginal distribution and which describes a conditional distribution?

d. Construct a joint distribution.

e. Find the fraction of people infected.

f. Find the covariance of age and infection, and explain why it is positive, negative or zero.

2.

a. Sketch and find the magnitude and direction of vectors with components (4,-3) starting from (1,-1) and (3,-4) starting from (-1,1).

b. Find the locations of the ends of these two vectors in terms of their length and direction.

3. In the days before the Ides of March, the confidence of Brutus and Caesar, denoted by B and C respectively, follow

$$\begin{aligned} B_{t+1} &= 1.2B_t - 0.2C_t \\ C_{t+1} &= 0.95C_t - 0.1B_t. \end{aligned}$$

These values are updated each day.

a. Explain the terms in this updating system.

b. Write it as a matrix.

c. If $B_0 = 1$ and $C_0 = 1$, find B_1, C_1, B_2 and C_2.

d. Find the two-day updating system.

e. What do you think will happen to B and C after many days?

4. A bird cleans itself fanatically when it has two lice. In particular, the probability that it gets a new louse when it has 0 or 1 is 0.4. It never gets 2 lice in one day or loses a louse when it has 1. When it has 2, it removes one with probability 0.2 and both with probability 0.3.

a. Draw a diagram illustrating what this bird does.

b. Write the matrix describing this process.

c. Find the equilibrium vector.

d. Find the average number of lice on this bird.