MATH 1180: Final, May 2, 2016 NAME: Fred Adler

Do all five problems, each worth 40 points. Make sure I can find both the answer and how you got it. You can use 4 sides of notes (2 pages front and back). No calculators. Extra credit problems are worth 1 point.

1. Antarctic krill have a staggeringly large population size. Suppose the population of small krill B and large krill K changes from year to year by the following rules:

- Large krill survive with probability 0.75,
- Small krill survive with probability 0.5, and of the survivors, half stay small and half grow to be large
- Large krill produce 3.0 small krill as babies
- Small krill have no babies.

a. Write this as an updating system.

b. Write this in terms of a matrix and vectors.

c. If the population started with 8.0×10^{15} small krill and 4.0×10^{15} large krill, what would it be after 1 year?

d. How would you find whether this population would grow or decline in the long run?

e. Change the equations to include some form of competition between krill, and explain what it means.

Extra credit: What single species of animal has the largest total mass?

a. L_{t+1} = surviving large krill + surviving small krill that grow $= 0.75 L_t + 0.25 S_t$

S_{t+1} = babies + surviving small krill that don't grow $= 3.0 L_t + 0.25 S_t$

b. $\begin{pmatrix} L_{t+1} \\ S_{t+1} \end{pmatrix} = \begin{pmatrix} 0.75 & 0.25 \\ 3.0 & 0.25 \end{pmatrix}\begin{pmatrix} L_t \\ S_t \end{pmatrix}$ & $\begin{pmatrix} S_{t+1} \\ L_{t+1} \end{pmatrix} = \begin{pmatrix} 0.25 & 3.0 \\ 0.25 & 0.75 \end{pmatrix}\begin{pmatrix} S_t \\ L_t \end{pmatrix}$

this is easier to read

c. $\begin{pmatrix} 0.25 & 3.0 \\ 0.25 & 0.75 \end{pmatrix}\begin{pmatrix} 8\times10^{15} \\ 4\times10^{15} \end{pmatrix} = \begin{pmatrix} 14\times10^{15} \\ 5\times10^{15} \end{pmatrix}$

d. I'd find the largest eigenvalue and check if it is greater than 1.

e. If large krill compete for food, the survival 0.75 would be a decreasing function of L_t, say

$L_{t+1} = 0.75 e^{-dL_t} L_t + 0.25 S_t$

2. Scientists are concerned about ill krill. Testing shows that krill are ill with probability 0.3 if they have been exposed to high temperature (HT), with probability 0.9 if they have been exposed to toxic red tide (TRT), and with probability 0.1 if they have been exposed to neither. 10% of krill are exposed to HT and 10% are exposed to TRT. No krill are exposed to both HT and TRT.

   a. Draw a table or chart illustrating these data.

   b. What is the probability that an ill krill has been exposed to TRT?

   c. Suppose we decided to use illness as a test for exposure to TRT. How many type I errors (false positives) and type II errors (false negatives) would this lead to?

Extra credit: What is one thing you learned about birds in this class?

a)

	HT	TRT	Neith	
ILL	0.03	0.09	0.08	→ 0.2
NOT ILL	0.07	0.01	0.72	→ 0.8
	↓ 0.1	↓ 0.1	↓ 0.8	

b) $\Pr(TRT/ILL) = \frac{0.09}{0.2} = 0.45$

c. Suppose we tested 100 krill

	ILL	NOT ILL
TRT	9 TRUE POSITIVE	1 FALSE NEGATIVE
NOT TRT	11 FALSE POSITIVE	79 TRUE NEGATIVE

We have a 0.11 false positive rate and a 0.01 false negative rate

3. Despite their staggeringly large population size, krill may be subject to reductions during El Niño years. In 4 El Niño years, the population averages 3.0×10^{15}, while in 12 non-El Niño years, it averages 4.0×10^{15}. The sample variance is 1.0×10^{30} for both sets of years.

   a. Sketch a graph of the probability density function of population size in an El Niño year.

   b. Sketch a graph of the probability density function of average population size during the four El Niño years.

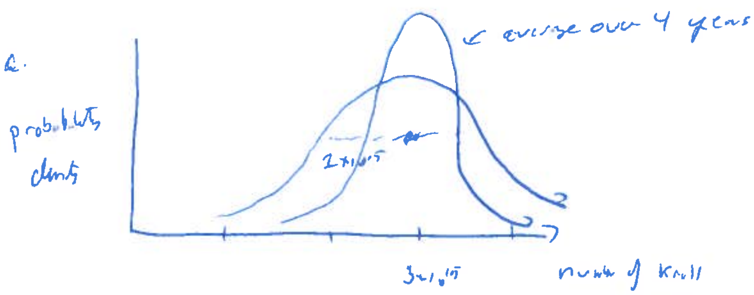
   c. Write the null hypothesis that El Niño years have no effect.

   d. Find the t-statistic.

   e. How many degrees of freedom (df) are there?

   f. Use the critical values of the t-distribution to check whether the difference is significant.

Extra credit: What is one thing you learned about, um, mathematical biology in this class?



df	p=0.05	p=0.01
3	3.182	5.841
4	2.776	4.604
9	2.262	3.250
11	2.262	3.250
12	2.179	3.055
13	2.160	3.012
14	2.145	2.977
15	2.131	2.947
16	2.120	2.921

b. The standard error is smaller by a factor of $\sqrt{4} = 2$

c. Let $\bar{E}$ = average in El Niño years, $\bar{N}$ = average in non El-Niño years.
Null hypothesis is that $\bar{E} - \bar{N} = 0$

d. The pooled variance is an average of the two sample variances, or 1.0×10^{30} because both are equal.
The standard error of the difference is $\sqrt{\frac{1.0\times10^{30}}{12} + \frac{1.0\times10^{30}}{4}} = \sqrt{\frac{1.0\times10^{30}}{3}} = 0.58\times10^{15}$

$$t = \frac{3.0\times10^{15} - 4.0\times10^{15}}{0.58\times10^{15}} \cong -1.73$$

e. $\nu = n_1 + n_2 - 2 = 4 + 12 - 2 = 14$

f. Nope, not too close to the critical value of 2.145

4. A whale-watching vessel waits in a krill swarm for 3.0 hours, recording the arrival time of whales, assumed to occur according to a Poisson process. The experiment starts at $t = 0$.

Whale	Arrival time
1	0.1
2	0.4
3	0.5
4	2.1
5	2.5

a. What distribution does the number of whales observed follow?

b. Write the likelihood function for the unknown parameter or parameters of this distribution, and find the maximum likelihood estimate.

c. What distribution does the waiting time between the arrival of whales follow?

d. Write the likelihood function for the unknown parameter or parameters of this distribution, and find the maximum likelihood estimate.

e. Do your answers to **b.** and **d** match? Explain why or why not.

Extra credit: What is your favorite number? Why?

a. The number of whales seen over the full three hours should follow a Poisson distribution

b. 5 whales were seen, so $\mathcal{L}(\Lambda) = \dfrac{\Lambda^5 e^{-\Lambda}}{5!}$

Then $S(\Lambda) = \ln(\mathcal{L}(\Lambda)) = 5\ln(\Lambda) - \Lambda - \ln(5!)$ $\quad \dfrac{dS}{d\Lambda} = \dfrac{5}{\Lambda} - 1 \Rightarrow \hat{\Lambda} = 5$

c. An exponential distribution

d. We have waiting times of 0.1, 0.3, 0.1, 1.6 & 0.4 between whales

Let λ be the parameter of the exponential.

$\mathcal{L}(\lambda) = (\lambda e^{-\lambda \cdot 0.1})(\lambda e^{-\lambda \cdot 0.3}) \cdot (\lambda e^{-\lambda \cdot 0.1})(\lambda e^{-\lambda \cdot 1.6}) \cdot (\lambda e^{-\lambda \cdot 0.4})$

$= \lambda^5 e^{-2.5\lambda}$

$S(\lambda) = \ln(\mathcal{L}(\lambda)) = 5\ln(\lambda) - 2.5\lambda, \quad S'(\lambda) = \dfrac{5}{\lambda} - 2.5 \Rightarrow \hat{\lambda} = 2.0$ whales/hour

e. This does not match $\hat{\Lambda} = 5$ whales in 3 hours.

I think it is because we didn't include that last half hour with no whales

5. The population of krill (in petakrill, or 10^{15} krill) was measured for six years using a highly accurate gravitational wave detector, which was then tragically destroyed by an angry albatross.

Year	Krill population	Model	e	Null model	ε_{null}
2005	5.5	5.0	0.5	4.5	1.0
2006	4.5	4.75	−0.25	4.5	0
2007	5.0	4.5	0.5	4.5	0.5
2008	4.0	4.25	−0.25	4.5	0.5
2009	4.5	4.0	0.5	4.5	0
2010	3.5	3.75	−0.25	4.5	1.0

a. Graph these data.

b. Find a line that passes through these data with $r^2 > 0$.

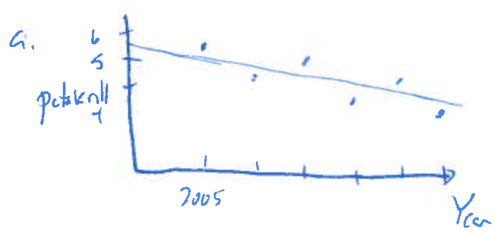
c. Find SSE for this line.

d. Find SST for these data.

e. Plot the residuals and describe whether they seem to match the assumptions of linear regression.

Extra credit: Write a really bad pun involving krill.

a.



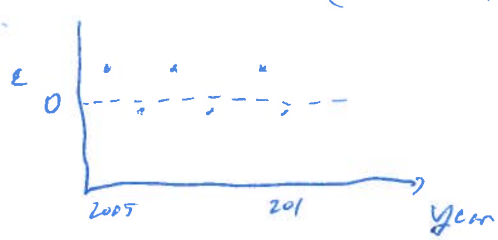
b. Goes down by about 0.25/yr, with an intercept of about 5.0 in 2005

c. $SSE = (0.5)^2 + (0.25)^2 + (0.5)^2 + (0.25)^2 + (0.5)^2 + (0.25)^2 = \frac{3}{4} + \frac{3}{16} = \frac{15}{16}$

d. To find SST we need $\bar{K} = \frac{5.5 + 4.5 + 5.0 + 4.0 + 4.5 + 3.5}{6} = \frac{27}{6} = 4.5$

$SST = 1^2 + 0^2 + (0.5)^2 + (0.5)^2 + 0^2 + 1.0^2 = 2.5$

e.



They don't look very normally distributed to me, and seem to alternate between above and below the line.