

Do all five problems, each worth 40 points. Make sure I can find both the answer and how you got it. Open textbook plus 2 pages of notes (each can be written on both front and back). No calculators. Each extra credit problem is worth 2 points.

1. People with swine flu arrive in a Gotham City at a rate of 1.5 per day, and recover at a per capita rate of 0.1 per day. There is no transmission between people, and infected people neither leave nor die.
 - a. Write a differential equation describing the number of infected people as a function of time.
 - b. What is the state variable in your equation?
 - c. Find the equilibrium or equilibria of your equation.
 - d. Find their stability.
 - e. Draw a phase line diagram.

Extra credit: What is the difference between a deterministic model and a stochastic model?

2. Untreated people are known to recover from swine flu at a probabilistic rate of 0.1 per day. Five people who are given tamiflu are observed to recover after 5, 6, 9, 7, and 8 days.
- What probability distribution describes the length of time a person is sick if untreated people really recover at rate 0.1 per day?
 - Find the likelihood and support functions if treated people recover at a constant but unknown rate of λ .
 - Find the maximum likelihood estimator of this recovery rate.
 - Draw a graph illustrating how you would use the method of support to check whether the observed recoveries in treated people are consistent with the known rate in untreated people. What number would you need to compute to do this?

Extra credit: What is $e^0 - \ln(2)$?

3. Anal (pronounced with the accent on the **second** syllable) City, with a population 1,000,000 people, establishes intense quarantine restrictions and ends up with 80 cases of swine flu. Gotham City, with 2,000,000 people, decides that the end of the world has come and does nothing. It ends up with 220 cases.
- What is the null hypothesis?
 - Find the expected number of infections in each city under the null hypothesis.
 - Find the chi-squared statistic and evaluate its significance.
 - Find the statistic G .
 - How do you interpret these results?

Extra credit: What would you call a virus that emerged from a cross between a male pig and a female dogote? How about a male dogote and a female pig?

4. 6 people in Anal City have an average recovery time of 6.0 days, while 9 people in Gotham City have an average recovery time of 15.0 days. The sample variances are 74.0 and 100.0 respectively.
- What is the null hypothesis?
 - Show that the pooled sample variance is 90.0.
 - Find the standard error of the mean.
 - Find the t value.
 - Is the difference significant with a two-tailed test?

Extra credit: What is calculus?

5. The government of Anal City implements a testing program even though they know there is a probability of $\alpha = 0.1$ of a false positive (a positive test even though a person is not infected) and a probability $\beta = 0.2$ of a false negative (a negative test even though a person is infected). The fraction of infected people is the unknown q . In a sample of 100 people, 83 tests come out negative and 17 come out positive.
- Find the total probability p that a test comes out positive as a function of q . Make sure to include both infected and uninfected people in this calculation.
 - What distribution describes the number of positive tests?
 - What is the maximum likelihood estimator of p ?
 - What is the maximum likelihood estimator of q ?
 - With these estimates, what is the probability that a person who tests positive is in fact infected?

Extra credit: If the derivative is positive, what do you know about the function?