Modeling the Dynamics of Life: Calculus and Probability for Life Scientists

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8.10 Supplementary Problems for Chapter 8

• EXERCISE **8.1**

Bacteria can be in one of two states: chemical producing (with probability 0.2) or chemical absorbing (with probability 0.8). Bacteria produce chemical at a rate of 2.0 femtomoles/second or absorb at a rate of 4.0 femtomoles/second. Suppose there are 100 bacteria in a culture, acting independently.

a. Find the mean and variance of the number in the producing and absorbing states.
b. Find the mean and variance of the rate of change of total chemical.
c. Write an exact expression for the probability that the amount of chemical is decreasing at any particular time. How would you evaluate it?
d. If instead of acting independently, all bacteria respond to the same external cue (but with the same probabilities), find the answers to parts **b** and **c**.

• EXERCISE **8.2**

It is observed that a population grows by a factor R determined by the temperature T as follows:

$$R = \begin{cases} 1.5 \text{ with probability } 0.2 \text{ if T=10} \\ 0.5 \text{ with probability } 0.8 \text{ if T=10} \\ 1.5 \text{ with probability } 0.7 \text{ if T=20} \\ 0.5 \text{ with probability } 0.3 \text{ if T=20} \end{cases}$$

Furthermore $T = 10$ with probability 0.4 and $T = 20$ with probability 0.6.

a. Will this population grow?
b. Find the correlation of temperature and growth rate.

• EXERCISE **8.3**

One team hits 30 out of 100 shots, and another hits 40 out of 100. Use the normal approximation to decide whether the second team really shoots better.

• EXERCISE **8.4**

Cosmic rays are thought to hit an object according to a Poisson process. 19 hit during the first minute and 25 during the second. Use the normal approximation to find 98% confidence limits around the maximum likelihood estimate of the true rate.

• EXERCISE **8.5**

Molecules bind to a certain type of receptor at a rate of λ per second and never unbind.

a. Suppose the receptor begins in an unbound state. Write and solve a differential equation for the probability the receptor is unbound at time t.
b. A cell has two of these receptors. One binds at time $t = 1.5$ and another at $t = 2.5$. Find the maximum likelihood estimate of λ.

• EXERCISE **8.6**

Suppose T measures temperature above 37°C in °C, and A measures an activity level. For three cells, $t_1 = 1$, $t_2 = 2$, $t_3 = 3$, $a_1 = 2$, $a_2 = 3$ and $a_3 = 3$

a. Plot A against T.
b. Find the linear regression of A on T.
c. Find the residuals.

• EXERCISE **8.7**

A model indicates that the probability a cell is healthy after a treatment is $1 - q$, that it is damaged is $2q/3$ and that it is moribund is $q/3$ for some unknown parameter q. 20 cells are tested: 10 are found to be healthy, 6 found to be damaged and 4 found to be moribund.

a. Find the likelihood function and the maximum likelihood estimate of q.

b. Without the model, the probabilities are $1 - q_1 - q_2$ (healthy), q_1 (damaged) and q_2 (moribund), with maximum likelihood estimates $q_1 = 0.3$ and $q_2 = 0.2$. Find the likelihood in this case. Do you think this model is better supported by the data than the one in **a**?

• EXERCISE **8.8**

Although students struggled heroically to correctly solve the 8 problems on their final, their professor assigned random grades independently for each problem and student, giving 10 with probability 0.2, 15 with probability 0.3, 20 with probability 0.4 and 25 with probability 0.1.

a. Find the mean and variance of the total score.
b. Find the probability that a student gets a perfect score.
c. Use the normal approximation to find the probability that a student scores above 140. What is the lowest score which scores in the top 25%?
d. Sketch the distribution of scores on this test.

• EXERCISE **8.9**

A new drug produces measurable reduction in a symptom in 75 out of 100 patients tested. An older drug produces measurable reduction in the symptom 65% of the time.

a. Use the normal approximation to find 99% confidence interval for the fraction of patients aided by the new drug.
b. Test the hypothesis that the new drug is no better than the old drug. What is the significance level? Make sure to say whether you used a one- or two-tailed test and why.

• EXERCISE **8.10**

The following data for temperature T and height H are measured:

T	H
10.0	12.0
12.0	13.0
14.0	15.0
16.0	17.0
18.0	20.0

A proposed regression line is $H = T + 1.0$.

a. Graph the data and proposed regression line.
b. Find the residuals.
c. Find SSE (with this line), SST and r^2.

• EXERCISE **8.11**

The average density of trout along a stream is 0.2 per meter.

a. Find the probability of 3 or more trout in 5 meters.
b. Find the expected number and coefficient of variation of the number of trout in 1 kilometer.
c. If the stream flows at 2 meters per second (and the trout don't swim), find the rate at which trout pass a given point.

• EXERCISE **8.12**

50 students from a standard calculus class have a normally distributed scores on a standardized test with mean 60 and 30 students from an innovative new calculus class score a mean of 63. The standard deviation of each and every student's score is known to be 5.0.

a. In mathematical language, give the null hypothesis that students from the reformed class did no better than those from the standard class.
b. What is the significance level of the test?

• EXERCISE **8.13**

The following data describe 10 measurements taken of height from control and treatment populations known to have normal distributions with standard deviation of 4.0. Test whether the treatment mean is different from the control mean. What is the significance level? What would you do if you did not know the standard deviation?

control	treatment
11.0	8.86
10.8	16.7
13.3	12.8
3.03	12.9
14.5	15.2
9.36	21.1
3.77	7.84
6.88	9.36
7.92	7.89
8.97	11.2

• EXERCISE **8.14**

A lazy scientist wants to estimate the rate at which wolves leave Yellowstone Park by waiting for the first one to leave. This happens after 3.0 months.

a. Find the maximum likelihood estimate of the rate.
b. Find 95% confidence limits around this estimate.
c. How would you use the method of support to approximate these confidence limits? Write the equation you would solve.
d. When is the most likely time for the second wolf to leave? The mean time?

• EXERCISE **8.15**

A company is testing a new insecticide. They run three experiments, measuring the number out of 100 that survive at three dosages.

experiment number	dosage (x)	number surviving (y)
1	18	3
2	15	4
3	9	5

A highly paid statistician finds the linear regression $y = 7 - 0.2x$.

a. Graph the data and the proposed regression line.
b. Find SSE, SST and r^2.
c. Write the equation (using the data in the table) that the statistician solved to find this line.
d. Do you think that the pesticide works better at the higher dose? How would you check this statistically?

• EXERCISE **8.16**

After application of a mutagen, five 100,000 base sequences of mitochondrial DNA have 24, 44, 29, 33 and 30 mutations respectively. Without mutagen, it is known that the mutation rate is 0.0002 per base.

a. Use the method of support to check whether the data are consistent with the null hypothesis.
b. Use the normal approximation to test the hypothesis that the mutagen increases the mutation rate. What does your significance level mean?
c. How would you test the null hypothesis with the Monte Carlo method?

• EXERCISE **8.17**

The National Park Service has begun an intensive study of the elk in Yellowstone Park. As a first step, they wish to estimate elk density. A preliminary survey locates 36 elk droppings in one square kilometer.

a. What is the sample and what is the population in this case?
b. What assumptions must you make to estimate the true density of elk droppings in the park?
c. What equations would you solve to find exact 99% confidence limits around an estimate of the density?
d. Use the normal distribution to find approximate 99% confidence limits.
e. How might you use these data to estimate the actual number of elk?

• EXERCISE **8.18**

To corroborate the intensive survey, elk dropping counters walk in straight lines at exactly 2 kilometers per hour, recording the first 10 times when they spot a dropping. The data from one surveyor are

dropping number i	time since last dropping (t_i)	dropping number i	time since last dropping (t_i)
1	0.124	6	0.118
2	0.056	7	0.220
3	0.244	8	0.014
4	0.227	9	0.015
5	0.001	10	0.016

An untrained but trustworthy assistant computes that

$$\begin{aligned}\sum_{i=1}^{10} t_i &= 1.035\\ \sum_{i=1}^{10} t_i^2 &= 0.193.\end{aligned}$$

a. Sketch a graph of where the droppings are.
b. What is your best guess of the rate at which droppings are encountered?
c. Write and sketch the support function.
d. Sketch how you would use this graph to find approximate confidence limits and write the equation you would solve to find them.
e. How might you relate these data to those in the previous problem?

• EXERCISE **8.19**

In a second stage in the project, managers decide to count both elk and their droppings. In 5 different square kilometer regions, they find

sample number i	number of droppings d_i	number of elk e_i
1	39	85
2	49	101
3	49	104
4	36	78
5	51	111

a. Graph these data.
b. A mathematician proposes two lines, $e_i = 2d_i + 5$ and $e_i = 5d_i + 2$. Sketch them. Which line gives a better fit to the data?
c. Find r^2 for the better of the two lines.
d. Interpret the relation described by this line. Does it make sense?

• EXERCISE **8.20**
In an even more advanced study, the managers use portable scales to weigh elk inside and outside the park to check whether they are different. They weigh 30 adult males inside the park (weights denoted by $x_1 \dots x_{30}$) and 40 adult males outside (weights denoted by $y_1 \dots y_{40}$), all in kilograms. They compute

$$\begin{aligned}
\sum_{i=1}^{30} x_i &= 1.829\times 10^4 \\
\sum_{i=1}^{40} y_i &= 2.316\times 10^4 \\
\sum_{i=1}^{30} x_i^2 &= 1.146\times 10^7 \\
\sum_{i=1}^{40} y_i^2 &= 1.368\times 10^7
\end{aligned}$$

a. Find the sample mean and sample variance for each group of elk.
b. Find the standard error for each.
c. State the null hypothesis mathematically.
d. Test the null hypothesis statistically.

• EXERCISE **8.21**
Having estimated numbers and weights, the managers begin working on health. They capture 40 elk and check them for parasites. They had reported to Congress that no more than 20% of the elk in the park were infested, but find that 13 out of their 40 are.

a. What is the probability that their report to Congress was true but they got unlucky? What does this have to do with significance levels?
b. What would a Congressman compute if she were using the popular method of support instead?
c. Give one clever excuse the managers could give for their result. How might they argue for an expensive follow-up study?

Answers

8.1.

a. The number that are producing has a binomial distribution with $n = 100$ and $p = 0.2$, so the mean is 20 and the variance is 16. The number that are absorbing has a binomial distribution with $n = 100$ and $p = 0.8$, so the mean is 80 and the variance is 16.

b. Let P be a random variable giving the number that are producing. The number absorbing is $100 - P$. The rate R is

$$R = 2.0P - 4.0(100 - P) = 6.0P - 400.$$

Then $\mathrm{E}(R) = 6.0\mathrm{E}(P) - 400 = -280$ and $\mathrm{Var}(R) = 6.0^2\mathrm{Var}(P) = 576$.

c. It is decreasing if $R < 0$ or if $6.0P - 400 < 0$ or if $P < 67$. I would evaluate it by finding the cumulative probability $B(66; 100, 0.2)$.

d. Either all are producing (with probability 0.2) or all are absorbing (with probability 0.8). In the first case, $R = 200$, and in the second $R = -400$. Then $\mathrm{E}(R) = 0.2{\cdot}200{+}0.8{\cdot}(-400) = 280$ as before. But $\mathrm{Var}(R) = 57600$, which is 100 times larger. The probability that the chemical is decreasing is 0.8.

8.2.

a.

$$\begin{aligned}
\Pr(R = 1.5) &= \Pr(R = 1.5|T = 10)\Pr(T = 10) \\
&\qquad + \Pr(R = 1.5|T = 20)\Pr(T = 20) = 0.5 \\
\Pr(R = 0.5) &= \Pr(R = 0.5|T = 10)\Pr(T = 10) \\
&\qquad + \Pr(R = 0.5|T = 20)\Pr(T = 20) = 0.5.
\end{aligned}$$

The population will grow if the geometric mean is greater than 1. But

$$e^{\mathrm{E}(\ln(R))} = e^{\ln(1.5)\cdot 0.5+\ln(0.5)\cdot 0.5} = 0.866.$$

Alternatively, the arithmetic-geometric inequality guarantees that the geometric mean is less than 1 because the arithmetic mean is equal to 1.

b. $\mathrm{E}(R) = 1$, $\mathrm{E}(T) = 16$, $\mathrm{Var}(R) = 0.25$, $\mathrm{Var}(T) = 24$. The joint distribution is

	$R = 0.5$	$R = 1.5$
$T = 10$	0.32	0.08
$T = 20$	0.18	0.42

Therefore,

$$\mathrm{E}(RT) = 5 \cdot 0.32 + 15 \cdot 0.08 + 10 \cdot 0.18 + 30 \cdot 0.42 = 17.2$$

Then $\mathrm{Cov}(R,T) = 17.2 - 1 \cdot 16 = 1.2$, and the correlation is

$$\frac{\mathrm{Cov}(R,T)}{\sqrt{\mathrm{Var}(R)\mathrm{Var}(T)}} = \frac{1.2}{\sqrt{6}} = 0.490.$$

8.3. Let P_1 and P_2 be random variables representing the fraction of shots hit by each team. The variance is approximately $0.3 \cdot 0.7/100 = 0.0021$ for the first and $0.4 \cdot 0.6/100 = 0.0024$ for the second. Then P_1 has distribution approximately $N(p_1, 0.0021)$ and P_2 has distribution approximately $N(p_2, 0.0024)$. Therefore $D = P_1 - P_2$ has distribution $N(p_1 - p_2, 0.0045)$. Under the null hypothesis, $p_1 = p_2$, giving D distribution $N(0, 0.0045)$. We use a one-tailed test because we suspected that the second team was better.

$$\begin{aligned} \Pr(D \geq 0.1) &= \Pr\left(\frac{D}{\sqrt{0.0045}} \geq \frac{0.1}{\sqrt{0.0045}}\right) \\ &= \Pr(Z \geq 1.49) = 0.068. \end{aligned}$$

This is not a significant result, meaning that this could well have happened by chance. Nonetheless, it would be heavily interpreted by sports pundits.

8.4. The likelihood is

$$L(\Lambda) = \frac{e^{-\Lambda}\Lambda^{19}}{19!}\frac{e^{-\Lambda}\Lambda^{25}}{25!}. = \frac{e^{-2\Lambda}\Lambda^{44}}{19! \cdot 25!}.$$

This has a maximum at $\Lambda = 22$. The distribution of hits in two minutes is approximately $N(44, 44)$. The 98% confidence limits are

$$44 - 2.33\sqrt{44} = 28.5, \quad 44 + 2.33\sqrt{44} = 59.5.$$

The number of hits in one minute is half, giving confidence limits of 14.2 to 29.7.

8.5.

a. Let $P(t)$ be the probability unbound at time t. Then $P(0) = 1$ and

$$\frac{dP}{dt} = -\lambda P$$

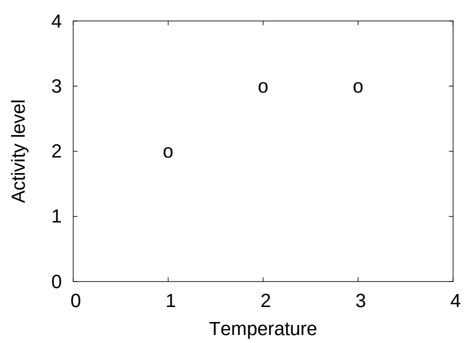
which has solution $P(t) = e^{-\lambda t}$.

b. The p.d.f. is $\lambda e^{-\lambda t}$. Then

$$L(\lambda) = \Pr(\mathrm{data}|\lambda) = \lambda e^{-1.5\lambda}\lambda e^{-2.5\lambda} = \lambda^2 e^{-4.0\lambda}$$

The maximum is at $\lambda = 0.5$.

8.6.



a.

b. $\widehat{\text{Var}}(T) = 0.67$ and $\widehat{\text{Cov}}(A, T) = 0.33$. Therefore

$$\hat{a} = \frac{\widehat{\text{Var}}(T)}{\widehat{\text{Cov}}(A, T)} = \frac{0.67}{0.33} = 0.5$$

Furthermore, $\bar{A} = 2.67$ and $\bar{T} = 2.0$, so

$$\hat{b} = \bar{A} - \hat{a}\bar{T} = 2.67 - 0.5 \cdot 2.0 = 1.67$$

The regression line is $\hat{A} = 0.5T + 1.67$.

c.

T	A	$\hat{A}$	$\hat{A} - A$
1	2	2.17	0.17
2	3	2.67	-0.33
3	3	3.17	0.17

8.7.

a.

$$
\begin{aligned}
L(q) &= (1-q)^{10}\left(\frac{2q}{3}\right)^6\left(\frac{q}{3}\right)^4 \\
&= \frac{2^6}{3^{10}}(1-q)^{10}q^6.
\end{aligned}
$$

This is maximized at $q = 0.5$.

b. The likelihood function is

$$
L_2(q_1, q_2) = (1 - q_1 - q_2)^{10} q_1^6 q_2^4.
$$

Taking logs, we find that the support $S_2(0.3, 0.2) = -20.59$. With the first model $S(0.5) = -20.69$. This is too small a difference to enable us to choose between the models.

8.8.

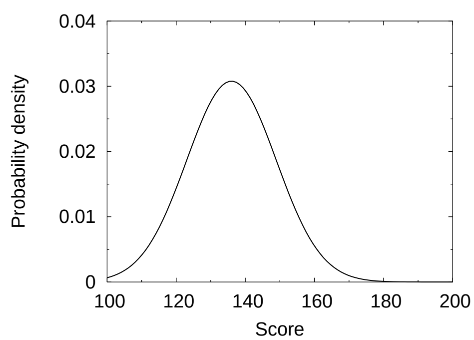
a. The mean for each problem is 17, and the variance is 21. The mean for the whole test is 136 and the variance is 168.

b. Assuming that 25 is perfect, the probability is $0.1^{10} = 10^{-10}$.

c. Scores have approximately a $N(136, 168)$ distribution.

$$
\Pr(S \geq 140) = \Pr\left(\frac{S-136}{\sqrt{168}} \geq \frac{140-136}{\sqrt{168}}\right) = \Pr(Z \geq 0.31) = 0.378.
$$

The lowest score in the top 25% is 0.675 standard deviations above the mean, or 145.



d.

8.9.

a. The fraction has distribution

$$N(\hat{p}, \frac{\hat{p}(1-\hat{p})}{100}) = N(0.75, 0.001875)$$

where $\hat{p} = 0.75$. The 99% confidence limits are

$$0.75 - 2.576\sqrt{0.001875} = 0.64, \quad 0.75 + 2.576\sqrt{0.001875} = 0.86$$

b. Under the null hypothesis, the fraction P has distribution

$$N(0.65, \frac{0.65(1-0.65)}{100}) = N(0.65, 0.00227).$$

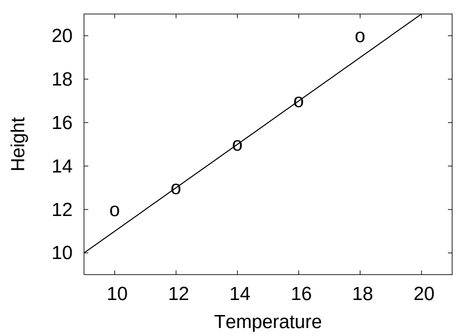
Then

$$\begin{aligned}\Pr(P \geq 0.75) &= \Pr\left(\frac{P-0.65}{\sqrt{0.00227}} \geq \frac{0.75-0.65}{\sqrt{0.00227}}\right)\\ &= \Pr(Z \geq 2.096) = 0.018.\end{aligned}$$

I used a one-tailed test because I have faith that new drugs are better, and found a significant result.

8.10.

a.



b.

T	H	$\hat{H}$	$\hat{H}-H$	$\bar{H}-H$
10.0	12.0	11.0	1.0	3.4
12.0	13.0	13.0	0.0	2.4
14.0	15.0	15.0	0.0	0.4
16.0	17.0	17.0	0.0	-1.6
18.0	20.0	19.0	-1.0	-4.6

c. SSE = 2.0, and SST=41.2. Therefore, $r^2 = 1 - \frac{2}{41.2} = 0.95$. The line is a pretty good fit (but not the best).

8.11.

a. The number of trout in 5 m has a Poisson distribution with $\Lambda = 0.2 \cdot 5 = 1$. Let T represent the number of trout.

$$\Pr(T \geq 3) = 1 - \Pr(T \leq 2) = 1 - p(0;1) - p(1;1) - p(2;1) = 0.08.$$

b. The expected number in 1000 m is 200; the coefficient of variation is then $\sqrt{200}/200 = 0.071$.

c. Multiply speed of stream by density of trout to get 0.4 trout per second.

8.12.

a. Let S_s represent the mean score of the standard class. It will have a $N(\mu_s, \frac{25}{\sqrt{30}})$ distribution. Let S_i represent the mean score of the innovative class. It will have a $N(\mu_i, \frac{25}{\sqrt{50}})$ distribution. The null hypothesis is that $\mu_s = \mu_i$, or that $\mu_s - \mu_i = 0$. The distribution of $D = S_s - S_i$ is $N(0, \frac{25}{\sqrt{30}} + \frac{25}{\sqrt{50}}) = N(0, 8.1)$ under the null hypothesis.

b. The difference in scores is 3.

$$\Pr(D \geq 3) = \Pr\left(\frac{D}{\sqrt{8.1}} \geq \frac{3}{\sqrt{8.1}}\right) = \Pr(Z \geq 1.05) = 0.146$$

The significance level is 0.146, meaning there is no reason to suspect that the innovative class really worked.

8.13. The mean of the control is 8.95 and of the treatment is 12.38. The null hypothesis is that the treatment came from distribution $N(8.95, 16/\sqrt{10}) = N(8.95, 5.06)$. Using a two-tailed test,

$$\begin{aligned}\Pr(H \geq 12.38) + \Pr(H \leq 5.52) &= \Pr\left(\frac{H - 8.95}{\sqrt{5.06}} \geq \frac{3.43}{\sqrt{5.06}}\right) + \Pr\left(\frac{H - 8.95}{\sqrt{5.06}} \leq \frac{-3.43}{\sqrt{5.06}}\right) \\ &= \Pr(Z \geq 1.52) + \Pr(Z \leq -1.52) = 0.128.\end{aligned}$$

Not significant. If I did not know the standard deviation, I would estimate it from the data, and then use a more advanced statistics book to figure out how to test the null hypothesis.

8.14.

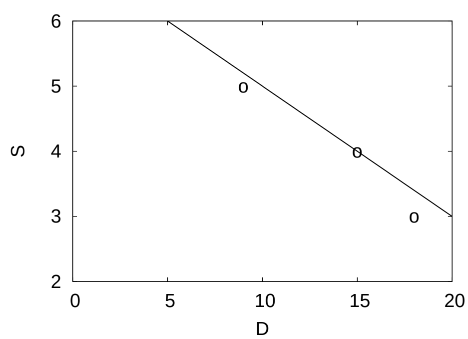
a. The likelihood function is $L(\lambda) = \lambda e^{-3\lambda}$. The maximum likelihood estimate is $\lambda = 0.33$.

b. For the lower limit, λ_l, solve $\Pr(T < 3|\lambda_l) = 0.025$ or $1 - e^{-3\lambda_l} = 0.025$, so $\lambda_l = 0.008$. For the upper limit, λ_h, solve $\Pr(T > 3|\lambda_h) = 0.025$ or $e^{-3\lambda_h} = 0.025$, so $\lambda_h = 1.23$.

c. I'd write $S(\lambda) = \ln(L(\lambda))$, and solve for the values of λ where $S(\lambda) = S(0.33) - 2$.

d. If the wolves leave independently, which is not likely, the most likely time for the second wolf is immediately after the first wolf. The mean time would be 3 months.

8.15.



a.

b.

D	S	$\hat{S}$	$\hat{S}-S$	$\bar{S}-S$
18.0	3.0	3.4	0.4	-1.0
15.0	4.0	4.0	0.0	0.0
9.0	5.0	5.2	0.2	1.0

Therefore, SST=2, SSE=0.2, $r^2 = 0.9$.

c. I have no idea what this statistician did other than incorrectly analyze an inadequate data set.

d. I'm not convinced it works. I would do more experiments.

8.16.

a. We would expect 20 mutations in each under the hypothesis that the mutagen does not work. The likelihood is

$$L(\Lambda) = \frac{e^{-\Lambda}\Lambda^{24}}{24!}\frac{e^{-\Lambda}\Lambda^{44}}{44!}\frac{e^{-\Lambda}\Lambda^{29}}{29!}\frac{e^{-\Lambda}\Lambda^{33}}{33!}\frac{e^{-\Lambda}\Lambda^{30}}{30!} = \frac{e^{-5\Lambda}\Lambda^{158}}{24!44!29!33!30!}$$

and the support is

$$S(\Lambda) = -5\Lambda + 158\ln(\Lambda) - \ln(24!44!29!33!30!)$$

The maximum likelihood estimate is 31.6, and

$$S(31.6) - S(20) = 14.27.$$

This looks like very strong support for the hypothesis that the mutagen increases mutation rates.

b. Under the null, we would get 100 total mutations in the 5 regions. The standard deviation is 10, so our data lie 5.8 standard deviations away. This is way off the chart, but can be found by computer to have a significance level of 3.3×10^{-9}.

c. I would take the known mutation rate, apply it to many 500,000 base sequences, and see how often I got 158 or more mutations.

8.17.

a. The sample is the elk droppings in one square kilometer. The population is all droppings in the park.

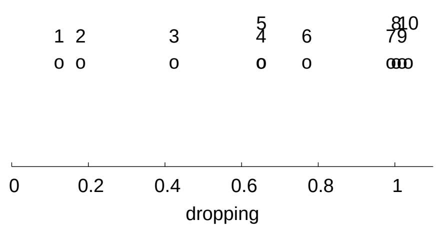
b. We must assume that the density of droppings is the same throughout the park.

c. The lower confidence limit is the largest value of k for which 36 or more elk droppings are found with probability less than 0.005. The upper confidence limit is the smallest value of k for which 36 or fewer elk droppings are found with probability less than 0.005.

d. With a mean and variance of 36, the confidence limits are 20.54 to 51.45.

e. Too disgusting to think about. Figure out number of droppings per elk, and how long they last.

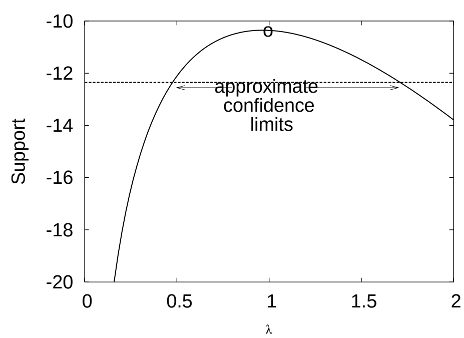
8.18.



a.

b. 0.966 per minute.

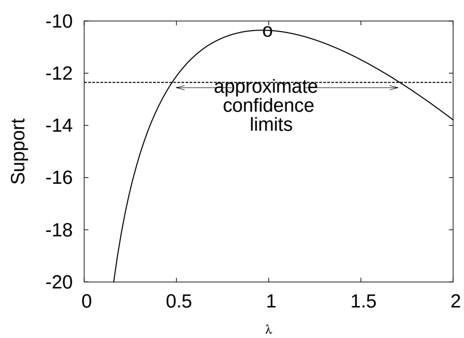
c. $S(\lambda) = 10\ln(\lambda) - 10\lambda \cdot 1.036.$



d. I would solve $S(\lambda) = S(0.966) - 2.0$.

e. Would have to figure out how fast people walked, and how wide an area they observe.

8.19.



a.

b. The first line gives a better fit.

c. SST=758.8, and SSE=26.0, so $r^2 = 0.966$.

d. The slope means that there is 1 elk for every 2 droppings. But I don't know what the 5 is for.

8.20.

a. $\bar{X} = 609.7$ with sample variance 10661. $\bar{Y} = 772.0$ with sample variance 27602.

b. The standard errors are 18.8 for X and 13.2 for Y.

c. The null hypothesis is that $\bar{X} = \bar{Y}$.

d. The means differ by 30.7. This is less than the total of the two standard errors, and is not significant.

8.21.

a. The probability of 13 or more is 0.0432. The value of $p = 0.2$ would lie just inside the 95% confidence limit (which allows for an error of 0.025.

b. The support for $p = 13/40$ is -25.22 and the support for $p = 0.2$ is -26.95. This is within 2, so $p = 0.2$ is reasonably well-supported by the data.

c. It might be easier to capture infected elk. They need to try different trapping methods and compare the results.