NAME:_____

MATH 1180
Midterm I

Do all three problems, points as indicated. Write readable answers on the test, but feel free to use or hand in additional paper if necessary. Remember, you can use one page of notes, but no calculator.

1. (30 points) A traffic jam of length L (measured in cars) is caused by a flipped Pepperidge Farm goldfish truck and follows the differential equation

$$\frac{dL}{dt} = b(1 - \frac{L}{K}) - aL$$

where t is measured in minutes and all parameters are positive.

a. The first term describes people joining the jam. What do b and K mean? What are their units?

b. The second term describes people leaving the road. What does a mean? What are its units?

c. Find the equilibrium or equilibria.

d. Draw the phase-line diagram for this equation.

e. Check stability with the stability theorem.

f. **Extra Credit:** Would a traffic jam be larger or smaller with a snack that smiles back?

2. (30 points) Suppose as in #1 that

$$\frac{dL}{dt} = b(1 - \frac{L}{K}) - aL$$

and that $b = 30$, $a = 0.2$, $K = 100$ and $L(0) = 10$.

a. Use Euler's method with step size $\Delta t = 1.0$ to estimate $L(1.0)$.

b. Show how you would use Euler's method for another step to estimate $L(2.0)$ (you don't have to work out the numbers).

c. Are these estimates higher or lower than the exact answer, and why?

d. Set up the equation you would use to find the exact solution algebraically and sketch a graph of that solution.

e. Extra Credit: What is your lab TA's first name? Last name?

3. (40 points) Suppose there are two lanes of traffic with L_1 and L_2 cars respectively, but people can exit only from lane 1, giving the equations

$$\begin{aligned} \frac{dL_1}{dt} &= b - sL_1 + sL_2 - aL_1 \\ \frac{dL_2}{dt} &= b - sL_2 + sL_1. \end{aligned}$$

Assume that all parameters are positive (overworked traffic planners left out the K terms for simplicity).

a. What do those s terms mean? Do they remind you of anything?

b. Draw the phase plane.

c. Add the nullclines.

d. Find the equilibria (you should get that $L_1^* = \frac{2b}{a}$).

e. Add the direction arrows.

f. Extra Credit: Do the s terms help reduce the size of the jam? What happens if $s = 0$?