

Do all five problems, each worth 40 points. Make sure I can find both the answer and how you got it. You can use 5 pages of notes (each can be written on both front and back). No calculators. Each extra credit problem is worth 2 points.

1. Over 20 years, a dedicated birdwatcher goes looking for Virginia's warblers in Dry Creek Canyon every May 1. He sees 0 warblers in one year, 1 warbler in twelve of the years, 2 warblers in five of the years, and 3 warblers in the remaining two years.
  - a. Why is a Poisson distribution is a good choice of distribution to describe these data? Why not?
  - b. Write the likelihood function for the parameter or parameters of the Poisson distribution using these data.
  - c. Find the maximum likelihood estimate(s).
  - d. How would you find the expected distribution with these estimates? How many years with 0 warblers would you expect?
  - e. Explain the steps you would use to test if the observed and expected data are different, illustrating with the years with 0 warblers. What problems could arise with this procedure?

**Extra credit:** What is one thing you learned about the history of statistics in this class?

number	value
$e^{0.7}$	2.0
$(0.7)^2$	0.5

2. A larger survey of Virginia's warblers was conducted in Parched Creek Canyon over the course of 25 years starting in 1930 with an average number seen of 20 birds. The survey was then repeated starting in 1985 for another 25 years with an average of 17 warblers seen per year.
- Suppose first that the number of birds found in a given year is known to be normally distributed with a mean of 20.0 and a variance of 36.0. What is the probability of seeing 17 or fewer warblers in one year? Sketch a graph illustrating this probability
  - Under the same assumptions as in **a.**, what is the probability of seeing an average of 17 or fewer warblers over the course of 25 years? What is the significance level? Sketch a graph illustrating this probability
  - Suppose instead that the sample variance was found to be 36.0 in both the first and second sets of 25 years. Explain the steps you would use to test whether the warbler populations had changed between the two surveys, including a clear statement of the null hypothesis.
  - What are two problems with this experiment?

**Extra credit:** What is one thing you learned about the history of mathematics in this class?

$z$	$\Phi(z)$
0.0	0.50000
0.5	0.69146
1.0	0.84134
1.5	0.93319
2.0	0.97725
2.5	0.99379
3.0	0.99865
3.5	0.99977
4.0	0.99997

3. Virginia's warblers migrate by catching the jet stream north from Mexico, which carries them horizontally at an altitude of 5000 meters. At the right moment above Arid Creek Canyon, they go into a nosedive, and begin to accelerate toward the ground at  $10 \text{ m/sec}^2$ .
- Write a differential equation describing the velocity of a nosediving warbler.
  - Solve this equation.
  - Write and solve a differential equation describing the altitude of a nosediving warbler.
  - Suppose that drag decreases the acceleration by an amount proportional to the velocity with constant of proportionality equal to 0.5. What are the units of this constant?
  - Write the differential equation for velocity that includes drag.
  - Find the equilibrium velocity including drag. Is this equilibrium stable?

**Extra credit:** What is one thing you learned about the history of music in this class?

4. Male Virginia's warblers sing to impress females. Some males have songs that are more melodious than others, measured in units of melodiocity. A study in Desiccated Creek Canyon found the following:

Male	Melodiocity	Number of females impressed
A	5	2
B	8	6
C	7	3
D	6	3
E	9	6

- What is the null hypothesis?
- Write a linear model for the relationship between melodiocity and the number of females impressed.
- Find a simple reasonable guess for the parameters of this model and graph it.
- Summarize the strength of this relationship in single statistic.
- What would you have to do to statistically test this relationship?

**Extra credit:** What is first thing that you learned in this class that you plan to forget?

5. People are worried that surveys are biased because singing birds are easier to locate than silent birds. This is tested by an intensive survey in Dusty Creek Canyon using highly trained Virginia's warbler-tracking flying dogs that find every single bird no matter what it is doing (but without harming or disturbing it in any way). It turns out that human observers locate 90% of singing birds but only 30% of silent birds, and that  $1/3$  of birds are singing at any one time.
- What is the probability that a bird that is located is singing?
  - Suppose these data had really been collected on 90 birds. Write the table of data that simultaneously describes whether birds were singing and whether they were located by people.
  - What would the table of data be under the null hypothesis that singing birds are no easier to locate than silent birds?

**Extra credit:** What is one thing that you learned in this class that you plan to remember?

**6. Extra credit:** Before looking, guess the theme of this test: