Modeling the Dynamics of Life: Calculus and Probability for Life Scientists

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1.12 Supplementary Problems for Chapter 1

• EXERCISE **1.1**
Suppose you have a culture of bacteria, where the density of each bacterium is 2.0 g/cm^3.

a. If each bacterium is $5\mu\text{m} \times 5\mu\text{m} \times 20\mu\text{m}$ in size, find the number of bacteria if their total mass is 30 grams. Recall that $1\mu\text{m} = 10^{-6}$ meters.
b. Suppose that you learn that the sizes of bacteria range from $4\mu\text{m} \times 5\mu\text{m} \times 15\mu\text{m}$ to $5\mu\text{m} \times 6\mu\text{m} \times 25\mu\text{m}$. What is the range of the possible number of bacteria making up the total mass of 30 grams?

• EXERCISE **1.2**
Suppose the number of bacteria in culture is a linear function of time.

a. If there are 2.0×10^8 bacteria in your lab at 5 p.m. on Tuesday, and 5.0×10^8 bacteria the next morning at 9 a.m., find the equation of the line describing the number of bacteria in your culture as a function of time.
b. At what time will your culture have 1.1×10^9 bacteria?
c. The lab across the hall also has a bacterial culture where the number of bacteria is a linear function of time. If they have 2.0×10^8 bacteria at 5 p.m. on Tuesday, and 3.4×10^8 bacteria the next morning at 9 a.m., when will your culture have twice as many bacteria as theirs?

• EXERCISE **1.3**
Consider the functions $f(x) = e^{-2x}$ and $g(x) = x^3 + 1$.

a. Find the inverses of f and g, and use these to find when $f(x) = 2$ and when $g(x) = 2$.
b. Find $f \circ g$ and $g \circ f$ and evaluate each at $x = 2$.
c. Find the inverse of $g \circ f$. What is the domain of this function?

• EXERCISE **1.4**
A lab has a culture of a new kind of bacteria where each individual takes two hours to split into 3 bacteria. Suppose that these bacteria never die and that all offspring are OK.

a. Write an updating function describing this system.
b. Suppose there are 2.0×10^7 bacteria at 9 a.m. How many will there be at 5 p.m.?
c. Write an equation for how many bacteria there are as a function of how long the culture has been running.
d. When will this population reach 10^9?

• EXERCISE **1.5**
The number of bacteria (in millions) in a lab are

Time (t)	Number (b_t)
0.0 hours	1.5
1.0 hours	3.0
2.0 hours	4.5
3.0 hours	5.0
4.0 hours	7.5
5.0 hours	9.0

a. Graph these points.
b. Find the line connecting them and the time t at which the value does not lie on the line.
c. Find the equation of the line and use it to find what the value at t would have to be to lie on the line.
d. How many bacteria would you expect at time 7.0 hours?

• EXERCISE **1.6**
The number of bacteria in another lab follows the discrete-time dynamical system

$$b_{t+1} = \begin{cases} 2.0 b_t & b_t \le 1.0 \\ -0.5(b_t - 1.0) + 2.0 & b_t > 1.0 \end{cases}$$

where t is measured in hours and b_t in millions of bacteria.

a. Graph the updating function. For what values of b_t does it make sense?
b. Find the equilibrium
c. Cobweb starting from $b_0 = 0.4$ million bacteria. What do you think happens to this population?

• EXERCISE **1.7**
Convert the following angles from degrees to radians and find the sine and cosine of each. Plot the related point both on a circle and on a graph of the sine or cosine.

a. $\theta = 60°$.
b. $\theta = -60°$.
c. $\theta = 110°$.
d. $\theta = -190°$.
e. $\theta = 1160°$.

• EXERCISE **1.8**
Suppose the temperature H of a bird follows the equation

$$H = 38.0 + 3.0 \cos(\frac{2\pi(t - 0.4)}{1.2})$$

where t is measured in days and H is measured in degrees C.

a. Sketch a graph of the temperature of this bird.
b. Write the equation if the period changes to 1.1 days. Sketch a graph.
c. Write the equation if the amplitude increases to 3.5 degrees. Sketch a graph.
d. Write the equation if the average decreases to 37.5 degrees. Sketch a graph.

• EXERCISE **1.9**
The butterflies on a particular island are not doing too well. Each autumn, every butterfly produces on average 1.2 eggs and then dies. Half of these eggs survive the winter and produce new butterflies by late summer. At this time, 1000 butterflies arrive from the mainland to escape overcrowding.

a. Write a discrete-time dynamical system for the population on this island.
b. Graph the updating function and cobweb starting from 1000.
c. Find the equilibrium number of butterflies.

• EXERCISE **1.10**
A culture of bacteria has mass 3.0×10^{-3} grams and consists of spherical cells of mass 2.0×10^{-10} grams and density 1.5 grams/cm^3.

a. How many bacteria are in the culture?
b. What is the radius of each bacterium?
c. If the bacteria were mashed into mush, how much volume would they take up?

• EXERCISE **1.11**
A person develops a small liver tumor. It grows according to

$$S(t) = S(0)e^{\alpha t}$$

where $S(0) = 1.0$ gram and $\alpha = 0.1$/day. At time $t = 30$ days, the tumor is detected and treatment begins. The size of the tumor then decreases linearly with slope of -0.4 grams/day.

a. Write the equation for tumor size at $t = 30$.
b. Sketch a graph of the size of the tumor over time.
c. When will the tumor disappear completely?

• EXERCISE **1.12**
Two similar objects are left to cool for one hour. One starts at 80°C and cools to 70°C and the other starts at 60°C and cools to 55°C. Suppose the discrete-time dynamical system for cooling objects is linear.

a. Find the discrete-time dynamical system. Find the temperature of the first object after two hours. Find the temperature after one hour of an object starting at 20°C.
b. Graph the updating function and cobweb starting from 80°C.
c. Find the equilibrium. Explain in words what the equilibrium means.

• EXERCISE **1.13**
A culture of bacteria increases in area by 10% each hour. Suppose the area is 2.0 cm^2 at 2:00 p.m.

a. What will the area be at 5:00 p.m.?
b. Write the relevant discrete-time dynamical system and cobweb starting from 2.0.
c. What was the area at 1:00 p.m.?
d. If all bacteria are the same size and each adult produces 2 offspring each hour, what fraction of offspring must survive?
e. If the culture medium is only 10 cm^2 in size, when will it be full?

• EXERCISE **1.14**
Candidates Dewey and Howe are competing for fickle voters. 100,000 people are registered to vote in the election and each will vote for one of these two candidates. Each week, some voters switch their allegiance. 20% of Dewey's supporters switch to Howe each week. Howe's supporters are more likely to switch when Dewey is doing well: the fraction switching from Howe to Dewey is proportional to Dewey's percentage of the vote – none switch if Dewey commands 0% of the vote and 50% switch if Dewey commands 100% of the vote. Suppose Howe starts with 90% of the vote.

a. Find the number of votes Dewey and Howe have after a week.
b. Find Dewey's percentage after a week.
c. Find the discrete-time dynamical system describing Dewey's percentage.
d. Graph the updating function and find the equilibrium or equilibria.
e. Who will win the election?

• EXERCISE **1.15**
A certain bacterial population has the following odd behavior. If the population is less than 1.5×10^8 in a given generation, each bacterium produces 2 offspring. If the population is greater than or equal to 1.5×10^8 in a given generation, it will be exactly 1.0×10^8 in the next.

a. Cobweb starting from an initial population of 10^7.
b. Graph a solution starting from an initial population of 10^7.
c. Find the equilibrium or equilibria of this population.

• EXERCISE **1.16**
An organism is breathing a chemical that modifies the depth of its breaths. In particular, suppose that the fraction q of air exchanged is given by

$$q = \frac{c_t}{c_t + \gamma}$$

where γ is the ambient concentration and c_t is the concentration in the lung. After a breath, a fraction q of the air came from outside, and a fraction $1 - q$ remained from inside. Suppose $\gamma = 0.5$ moles/liter.

a. Describe in words the breathing of this organism.
b. Find the discrete-time dynamical system for the concentration in the lung.
c. Find the equilibrium or equilibria.

• EXERCISE **1.17**
Lint is building up in a dryer. With each use, the old amount of lint x_t is divided by $1 + x_t$ and 0.5 lintons (the units of lint) are added.

a. Find the discrete-time dynamical system and graph the updating function.
b. Cobweb starting from $x_0 = 0$. Graph the associated solution.
c. Find the equilibrium or equilibria.

• EXERCISE **1.18**
Suppose people in a bank are waiting in two separate lines. Each minute several things happen: some people are served, some people join the lines, and some people switch lines. In particular, suppose that 1/10 of the people in the first line are served, and 3/10 of the people in the second line are served. Suppose the number of people who join each line is equal to 1/10 of the total number of people in both lines, and that 1/10 of the people in each line switch to the other.

a. Suppose there are 100 people in each line at the beginning of a minute. Find how many people are in each line at the end of the minute.
b. Write a discrete-time dynamical system for the number of people in the first line, and another discrete-time dynamical system for the number of people in the second.
c. Write a discrete-time dynamical system for the fraction of people in the first line.

• EXERCISE **1.19**
A gambler faces off against a small casino. She begins with \$1000, and the casino with \$11,000. In each round, the gambler loses 10% of her current funds to the casino and the casino loses 2% of its current funds to the gambler.

a. Find the amount of money each has after one round.
b. Find a discrete-time dynamical system for the amount of money the gambler has and another for the amount of money the casino has.
c. Find the discrete-time dynamical system for the fraction p of money the gambler has.
d. Find the equilibrium fraction of the money held by the gambler.
e. Using the fact that the total amount of money is constant, find the equilibrium amount of money held by the gambler.

• EXERCISE **1.20**
Let V represent the volume of a lung and c the concentration of some chemical inside. Suppose the internal surface area is proportional to volume, and a lung with volume 400 cm^3 has a surface area of 100 cm^2. The lung absorbs the chemical at a rate per unit surface area of

$$R = \alpha \frac{c}{4.0\times10^{-2} + c}.$$

Time is measured in seconds, surface area in cm^2 and volume in cm^3. The parameter α takes on the value 6.0 in the appropriate units.

a. Find surface area as a function of volume. Make sure your dimensions make sense.
b. What are the units of R? What must be the units of α?
c. Suppose that $c = 1.0\times10^{-2}$ $V = 400$. Find the total amount of chemical absorbed.
d. Suppose that $c = 1.0\times10^{-2}$. Find the total chemical absorbed as a function of V.

• EXERCISE **1.21**
Suppose a person's head diameter D and height H grow according to

$$\begin{aligned} D(t) &= 10.0e^{0.03t} \\ H(t) &= 50.0e^{0.09t} \end{aligned}$$

during the first 15 yr of life.

a. Find D and H at $t = 0$, $t = 7.5$ and $t = 15$.
b. Sketch graphs of these two measurements as functions of time.
c. Sketch semilog graphs of these two measurements as functions of time.
d. Find the doubling time of each measurement.

• EXERCISE **1.22**
On another planet, people have three hands and like to compute tripling times instead of doubling times.

a. Suppose a population follows the equation $b(t) = 3.0 \times 10^3 e^{0.333t}$ where t is measured in hours. Find the tripling time.
b. Suppose a population has a tripling time of 33 hours. Find the equation for population size $b(t)$ if $b(0) = 3.0 \times 10^3$.

• EXERCISE **1.23**
A Texas millionaire (with $1,000,001 in assets in 1995) got rich by clever investments. She managed to earn 10% interest per year for the last 20 years, and plans to do the same in the future.

a. How much did she have in 1975?
b. When will she have $5,000,001?
c. Write the discrete-time dynamical system and graph the updating function.
d. Write and graph the solution.

• EXERCISE **1.24**
A major university hires a famous Texas millionaire to manage its endowment. The millionaire decides to follow this plan each year:

- Spend 25% of all funds above $100 million on University operations.
- Invest the remainder at 10% interest.
- Collect $50 million in donations from wealthy alumni.

a. Suppose the endowment has $340 million to start. How much will it have after spending on University operations? After collecting interest on the remainder? After the donations roll in?
b. Find the discrete-time dynamical system.
c. Graph the updating function and cobweb starting from $340 million.

• EXERCISE **1.25**
Another major university hires a different famous Texas millionaire to manage its endowment. This millionaire starts with $340 million, brings back $355 million the next year, and claims to be able to guarantee a linear increase in funds thereafter.

a. How much money will this University have after 8 yr?
b. Graph the endowment as a function of time.
c. Write the discrete-time dynamical system, graph and cobweb starting from $340 million.
d. Which University do you think will do better in the long run? Which Texan would you hire?

• EXERCISE **1.26**
A heart receives a signal to beat every second. If the voltage when the signal arrives is below 50 microvolts, the heart beats and increases its voltage by 30 microvolts. If the voltage when the signal arrives is greater than 50 microvolts, the heart does not beat and the voltage does not change. The voltage of the heart decreases by 25% between beats in either case.

a. Suppose the voltage of the heart is 40 microvolts right after one signal arrives. What is the voltage before the next signal and will the heart beat?
b. Graph the updating function for the voltage of this heart.
c. Will this heart exhibit normal beating or some sort of AV block?

• EXERCISE **1.27**

Suppose vehicles are moving at 72 kph (kilometers per hour). Each car carries an average of 1.5 people, and all are carefully keeping a two second following distance (getting no closer than the distance a car travels in two seconds) on a 3 lane highway.

a. How far between vehicles?
b. How many vehicles per kilometer?
c. How many people will pass a given point in an hour?
d. If commuter number oscillates between this maximum (at 8:00 a.m.) and a minimum that is one third as large (at 8:00 p.m.) on a 24 hour cycle, give a formula for the number of people passing the given point as a function of time of day.

• EXERCISE **1.28**

Suppose traffic volume on a particular road has followed

Year	Vehicles
1970	40,000
1980	60,000
1990	90,000
2000	135,000

a. Sketch a graph of traffic over time.
b. Find the discrete-time dynamical system that describes this traffic.
c. What was the traffic volume in 1960?
d. Give a formula for the predicted traffic in the year 2050.
e. Find the half-life or doubling-time of traffic.

• EXERCISE **1.29**

In order to improve both the economy and quality of life, policies are designed to encourage growth and decrease traffic flow. In particular, the number of vehicles is encouraged to increase by a factor of 1.6 over each ten year period, but the commuters from 10,000 vehicles are to choose to ride comfortable new trains instead of driving.

a. If there were 40,000 vehicles in 1970, how many would there be in 1980?
b. Find the discrete-time dynamical system describing this traffic.
c. Find the equilibrium.
d. Graph the updating function and cobweb starting from an initial number of 40,000.
e. In the long run, will there be more or less traffic with this policy than with the policy that led to the data in the previous problem? Why?

Chapter 9

Answers

1.1. **a.** 3.0×10^{10} bacteria. **b.** $2.0\times 10^{10} \le \text{number} \le 5.0\times 10^{10}$.

1.2.

a. If B represents the number of bacteria and t the time in hours, $B = 2.0\times 10^7 + 3.0\times 10^7 t$.

b. 36 hours.

c. Their culture satisfies $B = 2.0\times 10^7 + 2.0\times 10^7 t$. Must solve for when $2.0\times 10^7 + 3.0\times 10^7 t = 2(2.0\times 10^7 + 2.0\times 10^7 t)$. Solution is $t = -2$ hours, which should be 3 p.m. on Tuesday. But at this time, both populations would have been negative. There is no biologically reasonable solution.

1.3.

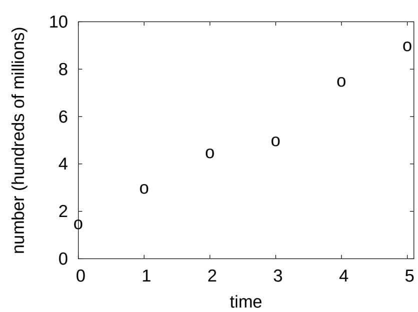
a. $f^{-1}(y) = -\ln(y)/2$, $g^{-1}(y) = \sqrt[3]{y-1}$. $f(x) = 2$ when $x = f^{-1}(2) = -\dfrac{\ln(2)}{2} \approx -0.345$. Similarly, we find $g(1) = 2$.

b. $(f \circ g)(x) = e^{-2(x^3+1)}$, $(g \circ f)(x) = e^{-6x} + 1$, $(f \circ g)(2) = e^{-18} = 1.5\times 10^{-8}$, $(g \circ f)(2) = 1 + e^{-12} = 1.00006$.

c. $(g \circ f)^{-1}(y) = -\ln(y-1)/6$, domain is $y > 1$.

1.4. **a.** $f(b) = 3b$. **b.** 1.62×10^9. **c.** $B(t) = 2.0\times 10^7 \times 3^{t/2}$, t measured in hours. **d.** $t = \ln(B/2.0\times 10^7)/\ln(3)$.

1.5.



a.

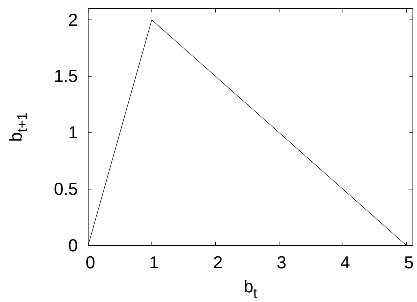
b. Line has slope of $1.5 \dfrac{\text{million bacteria}}{\text{hour}}$ and equation $b(t) = 1.5 + 1.5t$. At $t = 3$, the point 5.0 lies below the line.

c. Substituting $t = 3$ into the equation, we get 6.0 million.

d. Substituting $t = 7$ into the equation, we get 12.0 million.

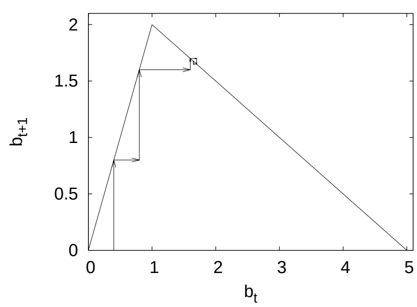
1.6.

a. This makes sense only for $b_t \leq 5.0$.



b. The equilibrium solves $b^* = -0.5(b^* - 1.0) + 2.0$ if $b^* > 1.0$. The solution is 1.66, which is indeed greater than 1.0..

c. It looks like the population approaches the equilibrium.



1.7.

a. $\pi/3$ radians. $\sin(\theta) = \sqrt{3}/2$, $\cos(\theta) = 1/2$.

b. $-\pi/3$ radians. $\sin(\theta) = -\sqrt{3}/2$, $\cos(\theta) = 1/2$.

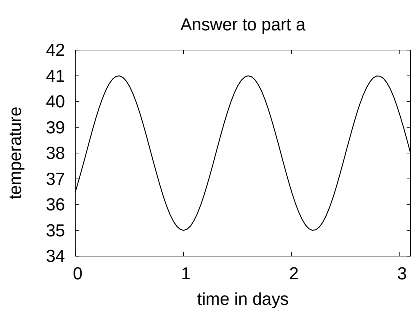
c. 1.919 radians. $\sin(\theta) \approx 0.9397$, $\cos(\theta) = -0.3420$.

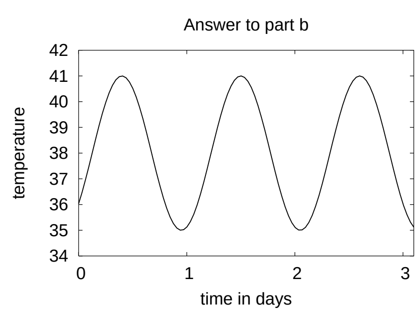
d. -3.316 radians. $\sin(\theta) \approx 0.1736$, $\cos(\theta) = -0.9848$.

e. 20.25 radians. $\sin(\theta) \approx 0.9848$, $\cos(\theta) = 0.1736$.

1.8.

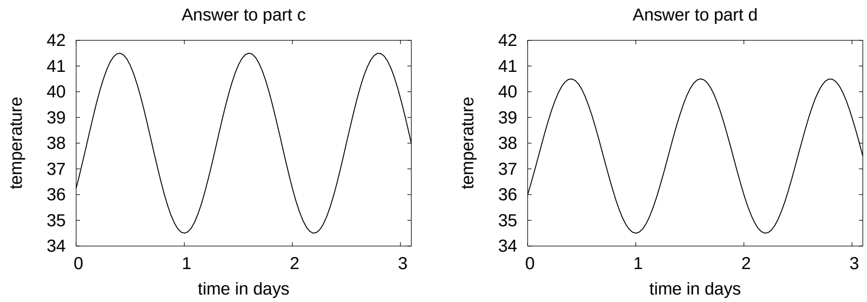
a.





b. $H = 38.0 + 3.0\cos\left(\dfrac{2\pi(t-0.4)}{1.1}\right)$

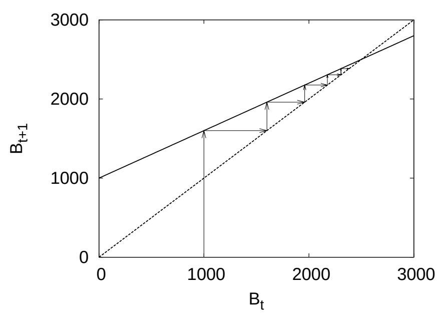
c. $H = 38.0 + 3.5\cos\left(\dfrac{2\pi(t-0.4)}{1.2}\right)$



d. $H = 37.5 + 3.0\cos(\dfrac{2\pi(t-0.4)}{1.2})$

1.9.

a. Let B_t be the number of butterflies in the late summer. There are then $1.2B_t$ eggs, leading to $0.6B_t$ new butterflies from reproduction plus 1000 from immigration. The discrete-time dynamical system is $B_{t+1} = 0.6B_t + 1000$.



b.

c. Equilibrium has 2500 butterflies.

1.10.

a. $M = N\mu$, so $N = M/\mu = 1.5\times10^7$.

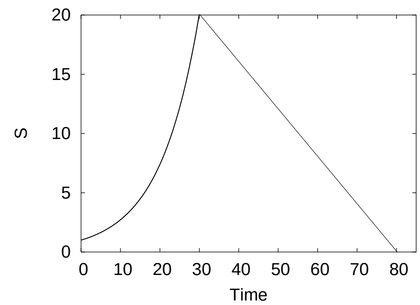
b. $V = \mu/\rho = 1.33\times10^{-10}\text{cm}^3 = \frac{4}{3}\pi r^3$. Therefore $r^3 = 10^{-10}/\pi$ and $r = 3.17\times10^{-4}\text{cm}$ or $317\mu\text{m}$.

c. The volume would be the same, $1.33\times10^{-10}\text{cm}^3$.

1.11.

a. The size at $t = 30$ is $S(30) = 1.0e^{0.1\cdot 30} \approx 20.08$. The size after treatment, which we can denote $S_T(t)$, is a line through the point $(30, 20.08)$ with slope -0.4, so

$$S_T(t) = -0.4(t-30) + 20.08.$$



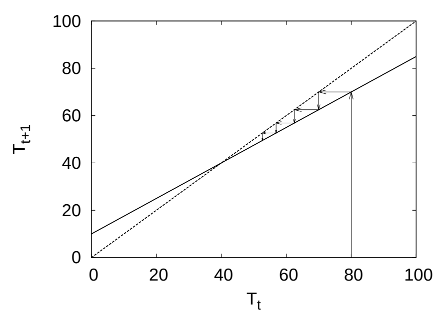
b.

c. We solve $S_T(t) = 0$, or

$$\begin{aligned} -0.4(t-30)+20.08 &= 0 \\ 0.4(t-30) &= 20.08 \\ t-30 &= \frac{20.08}{0.4} \\ t &= 30+\frac{20.08}{0.4} \approx 80.2. \end{aligned}$$

1.12.

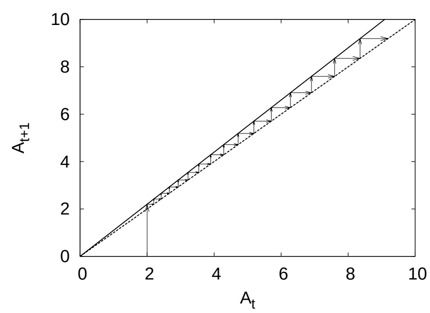
a. We have that $T_{t+1} = 70$ when $T_t = 80$ and that $T_{t+1} = 55$ when $T_t = 60$. The line connecting these points has equation $T_{t+1} = 0.75T_t + 10$. The temperature of the first object will be 62.5°C after two hours. An object starting at 20°C would warm up to 25°C in one hour.



b.

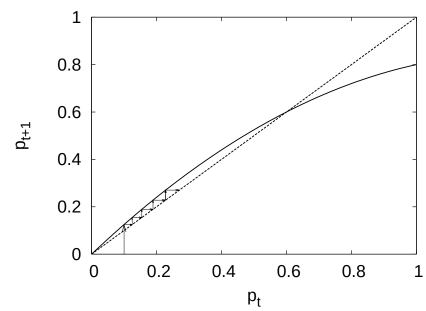
c. The equilibrium is $T^* = 40$°C. An object with this temperature will remain at the same temperature. This must be the temperature of the room.

1.13. **a.** 2.66cm^2. **b.** The discrete-time dynamical system is $A_{t+1} = 1.1A_t$. **c.** 1.82cm^2. **d.** 0.55. **e.** When $2.0 \times 1.1^t = 10$ or in 16.9 hours.



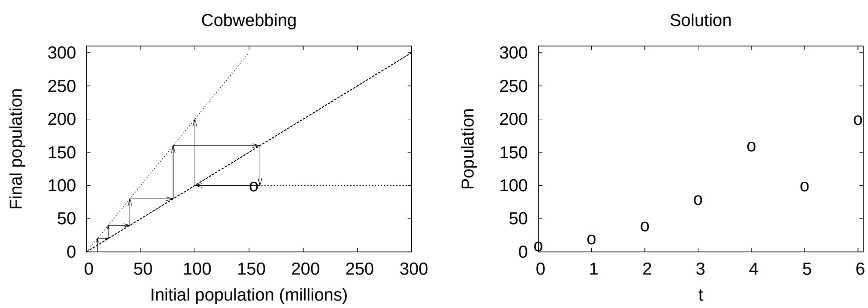
1.14.

a. Howe has 87,500 and Dewey has 12,500.
b. 12.5%.
c. $p_{t+1} = 0.8p_t + 0.5p_t(1 - p_t)$.
d. Equilibria are where $p_t = 0.8p_t + 0.5p_t(1 - p_t)$. This has solutions at $p_t = 0$ and $p_t = 0.6$.



e. It depends how long until the election. If it is held soon, Howe wins. After long enough, Dewey wins.

1.15.



b.

c. The only equilibrium is at 0.

1.16.

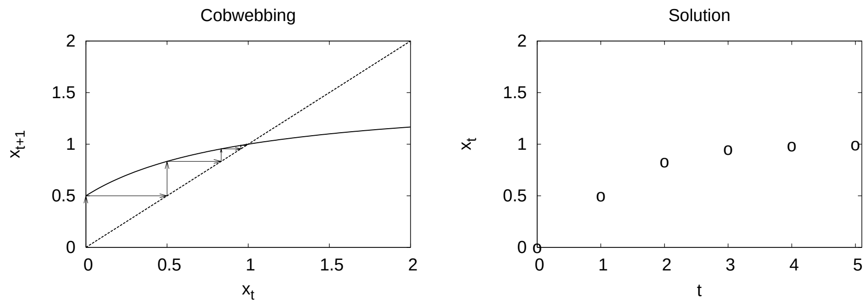
a. The higher the concentration of chemical, the deeper it breathes.
b.

$$c_{t+1} = (1 - q)c_t + q\gamma = (1 - \frac{c_t}{c_t + \gamma})c_t + \frac{c_t}{c_t + \gamma}\gamma = \frac{2\gamma c_t}{c_t + \gamma}.$$

c. $c_t = 0$ or $c_t = \gamma$.

1.17.

a. $x_{t+1} = 0.5 + x_t/(1 + x_t)$.



b. The equilibrium is at 1.0.

1.18.

a. First line: 10 people served, 20 join, 10 switch out, 10 switch in, so there are $100 - 10 + 20 - 10 + 10 = 110$ at the end of the minute. Second line: 30 people served, 20 join, 10 switch out, 10 switch in, so there are $100 - 30 + 20 - 10 + 10 = 90$ at the end of the minute.

b. Let a be the number in the first line and b the number in the second line. Then

$$a_{t+1} = 0.9a_t + 0.1(a_t + b_t) - 0.1a_t + 0.1b_t = 0.9a_t + 0.2b_t.$$

Similarly,

$$b_{t+1} = 0.2a_t + 0.7b_t.$$

c.

$$\begin{aligned} p_{t+1} &= \frac{a_{t+1}}{a_{t+1} + b_{t+1}} = \frac{0.9a_t + 0.2b_t}{1.1a_t + 0.9b_t} \\ &= \frac{0.9p_t + 0.2(1 - p_t)}{1.1p_t + 0.9(1 - p_t)} = \frac{0.7p_t + 0.2}{0.2p_t + 0.9} \end{aligned}$$

1.19.

a. She has \$1120, and the casino has \$10,880.

b. Let g represent the money the gambler has and c the amount the casino has. Then

$$\begin{aligned} g_{t+1} &= g_t - 0.1g_t + 0.02c_t = 0.9g_t + 0.02c_t \\ c_{t+1} &= c_t + 0.1g_t - 0.02c_t = 0.1g_t + 0.98c_t. \end{aligned}$$

c.

$$\begin{aligned} p_{t+1} &= \frac{g_{t+1}}{g_{t+1} + c_{t+1}} = \frac{0.9g_t + 0.02c_t}{g_t + c_t} \\ &= \frac{0.9p_t + 0.02(1 - p_t)}{p_t + (1 - p_t)} = 0.88p_t + 0.02. \end{aligned}$$

d. The equilibrium is $p^* = \frac{1}{6}$.

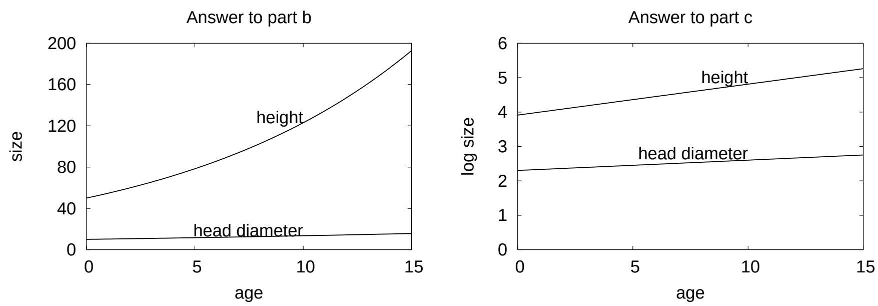
e. They start out with \$12,000, and the gambler ends up with 1/6, or \$2000.

1.20.

a. $S = \frac{V}{4}$.
b. Both R and α have units of moles per cm^2.
c. $R = 1.2\ \mathrm{moles/cm}^2$, so the total absorbed is $1.2{\times}10^2$ moles.
d. $T = 0.3V$.

1.21.

a. $D(0) = 10.0$, $H(0) = 10.0$, $D(7.5) = 12.5$, $H(7.5) = 98.2$, $D(15) = 15.7$, $H(15) = 193.9$.



d. Doubling time is 23.1 for head diameter and 7.7 for height.

1.22.

a. We need to solve for the time when $b(t) = 3b(0)$. The tripling time t_t is

$$t_t = \frac{\ln(3)}{0.333} \approx 3.30 \text{ hours.}$$

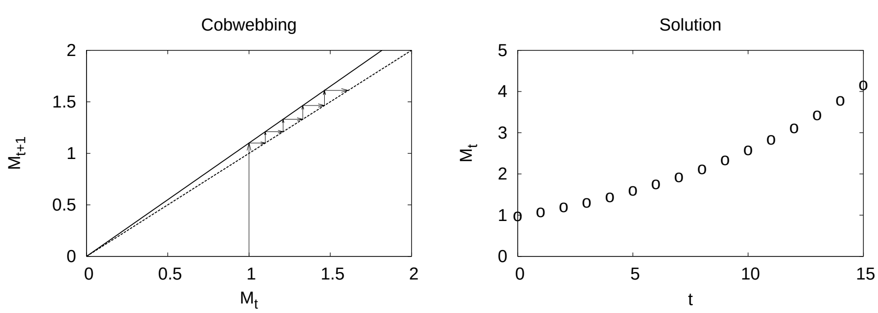
b. The value of α will be

$$\alpha = \frac{\ln(3)}{t_t} \approx 0.033.$$

The equation is $b(t) = 3.0{\times}10^3 e^{0.033t}$.

1.23.

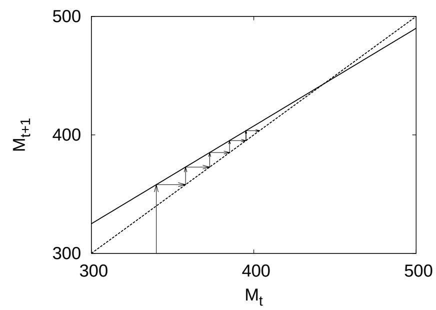
a. \$148,643.
b. In 16.88 yr, or in about 2012.
c. $M_{t+1} = 1.1M_t$.
d. $M_t = 1.000001{\times}10^6 \cdot 1.1^t$ where t is measured in years before or after 1995.



1.24.

a. Spend \$60 million on operations, leaving \$280 million. Get \$28 million in interest and \$50 million in donations for a total of \$358 million.

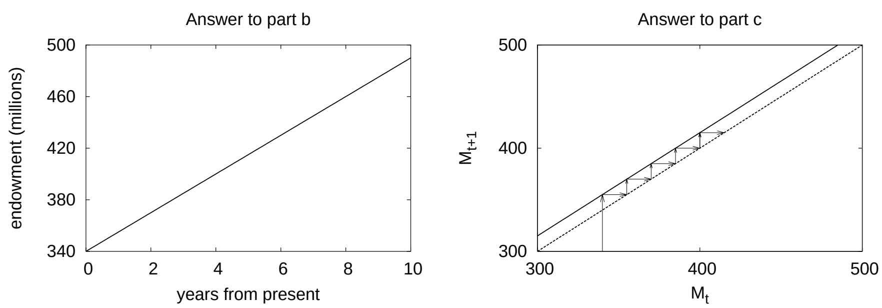
b. $M_{t+1} = 0.825M_t + 77.5$.



1.25.

a. It will have \$460 million.

b.



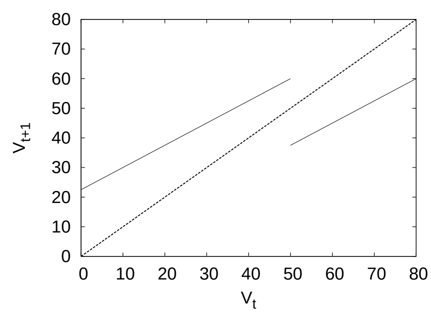
c. $M_{t+1} = M_t + 15$.

d. I'd hire the second Texan – the money keeps piling up.

1.26.

a. It decreases to 30, and it will beat.

b.



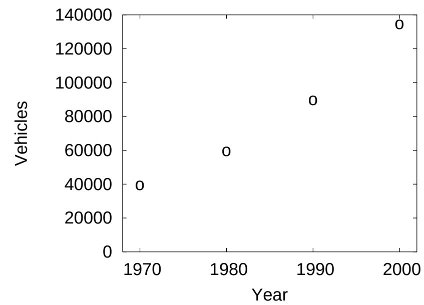
c. The equilibrium would be when $v^* = 0.75v^* + 30$ or $v^* = 120$ if it beat every time. But if it starts at 120, it decreases only to 90, too high to beat again. This heart will show some sort of AV block.

1.27.

a. The distance a car moves in 2 seconds is 40 meters.
b. There will be 25 vehicles per kilometer.
c. Then will be $25 \cdot 72 = 1800$ vehicles passing a point in one hour, carrying 2700 people.
d. This oscillation has period 24 hours, amplitude of 900 (half the difference between the minimum of 900 and the maximum of 2700), a mean of 1800, and a phase of 8.0, with formula

$$p(t) = 1800 + 900 \cos(\frac{t - 8.0}{24}).$$

1.28.



a.

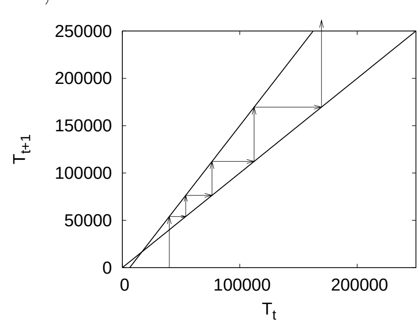
b. It is increasing by 50% per decade, so

$$T_{t+1} = 1.5 T_t$$

where T is traffic and t is time in decades.
c. We divide by 1.5 to find 26,667.
d. After 5 more decades, we would find $135,000 \cdot 1.5^5$ cars.
e. To find the doubling time, we write the solution in exponential form at $e^{\ln(1.5)t}$. The doubling time is $\ln(2)/\ln(1.5) \approx 1.71$ decades or 17.1 years.

1.29.

a. $40,000 \cdot 1.6 - 10,000 = 54,000$.
b. $T_{t+1} = 1.6 T_t - 10,000$.
c. Solving $T_{t+1} = T_t$ gives $T^* \approx 16,667$.



d.

e. This one will grow faster because traffic goes up by 60% rather than 50%. Once the numbers get really big, the 10,000 car reduction won't matter much.