NAME: Fred Adler

MATH 1170
Midterm III

Do all **four** problems (points as indicated). Write readable answers on the test, but feel free to use or hand in additional paper. You can use one page of notes, but no calculators or any quasi-intelligent device other than your brain.

1. (25 points) The total duration of chase scenes (in hours) during James Bond films follows the discrete-time dynamical system

$$C_{t+1} = \frac{2.0}{1 + C_t}$$

where t represents the current Bond film and $t + 1$ the next.

a. Find the positive equilibrium C^* (guessing is a good idea).

b. Find the stability of the equilibrium with the slope criterion.

c. Sketch a graph and cobweb starting from $C_0 = 0$.

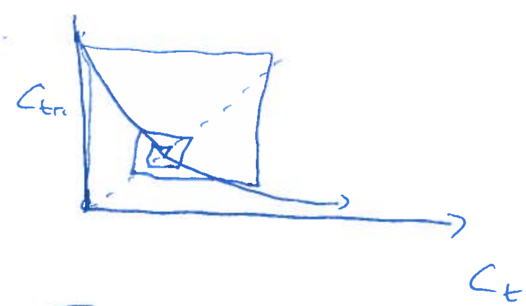
d. If $C_t = C^* - 0.0014$, use the slope at the equilibrium to find an approximate value of C_{t+1}.

a. I guess $c^* = 1$. Checking: $\frac{2.0}{1+1} = 1$. It worked!

b. Let $g(c) = \frac{2}{1+c}$. $g'(c) = \frac{-2}{(1+c)^2}$ so $g'(1) = \frac{-2}{(1+1)^2} = -\frac{2}{4} = -\frac{1}{2}$

This lies between -1 and 1 so it's stable.

c.



d. The distance from equilibrium decreased by an approximate factor of $g'(c^*) = -\frac{1}{2}$

If we were below by 0.0014 in one step, we'd be above by 0.0007 in the next.

$C_{t+1} \approx 1 + 0.0007 = 1.0007$

2. (25 points) Budget constraints have made it impossible for a single movie to have a total of more than 10 gadgets (G) and Bond girls (B) combined. Suppose that the opening weekend audience (in millions) is

$$A = 8\ln(G) + 2\ln(B)$$

a. Write the fact that the total is equal to 10 mathematically in terms of G and B.

b. Write A as a function only of G (use part **a** to eliminate B).

c. Find the number of gadgets that maximizes the audience.

d. Sketch a graph of this function over the reasonable domain of values for G.

a. $G + B = 10$

b. $B = 10 - G \Rightarrow A = 8\ln(G) + 2\ln(10-G)$

c. $\frac{dA}{dG} = \frac{8}{G} - \frac{2}{10-G}$ $\frac{dA}{dG} = 0$ if $\frac{8}{G} = \frac{2}{10-G}$

$8(10-G) = 2G$

$80 - 8G = 2G$, $80 = 10G$, $\boxed{G = 8}$

d. If $G = 1$, $A = 8\ln(1) + 2\ln(9) = 2\ln(9)$

If $G = 9$, $A = 8\ln(9) + 2\ln(1) = 8\ln(9)$

A is not defined if $G = 0$ or $G = 10$

3. (25 points) The lucky actor who plays James Bond receives a bonus (in thousands of dollars) based on the number of minutes t spent in the presence of poisonous arthropods during filming, according to the function

$$g(t) = t^{3.5} - 1.$$

a. How much bonus does the actor receive if $t = 1$?

b. Use the tangent line to approximate the bonus if $t = 1.002$. What could the actor purchase with this?

c. Show how you would use Taylor series to find a better approximation.

d. On a graph of $g(t)$, show how you could use Newton's method to find the value of t for which $g(t) = 1.0$.

a. $g(1) = 1^{3.5} - 1 = 0.$ No bonus

b. $g'(t) = 3.5t^{2.5}$, $g'(1) = 3.5$. So $g_t(t) = 0 + 3.5(t-1)$

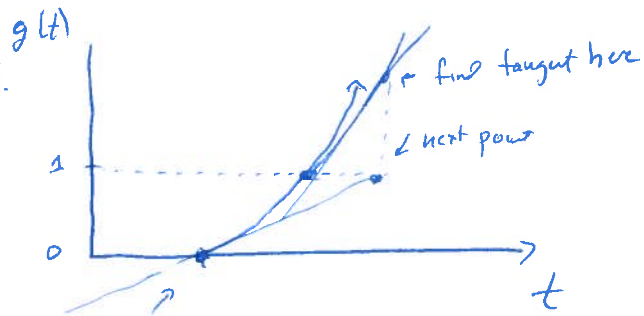
$g_t(1.002) = 0 + 3.5(0.002) = 0.007$

This is \$7. Ultra-grade Bond martini Latte

c. $g''(t) = 3.5 \cdot 2.5 \cdot t^{1.5}$, so $g''(1) = 3.5 \cdot 2.5 = 8.75$ $\quad g_2(t) = 0 + 3.5(t-1) + \frac{8.75}{2}(t-1)^2$

Could continue with higher derivatives

d.



a) start here

b) find where tangent line hits $g(t)=1$

c) Use this as next estimate

number	value
$2^{3.5}$	11.31
$3.5^{2.0}$	12.25
$3.5 \cdot 2.0$	7.00
$3.5 \cdot 4.5$	15.75
$3.5 \cdot 2.5$	8.75

4. (25 points) The cost C (in dollars) of making a Bond film and the cost of paying the animators A to make it all seem real increase according to

$$\begin{aligned} C(y) &= 1.0\times 10^6 e^{0.1y} + 0.007y^5 \\ A(y) &= 6.0\times 10^5 e^{-0.007y} + 0.007y^6 \end{aligned}$$

where y is in years starting from May 8, 1963.

a. Find the leading behavior of $C(y)$ as $y \to \infty$.

b. Find the leading behavior of $A(y)$ as $y \to 0$.

c. What is $\lim_{y\to\infty} A(y)/C(y)$?

d. How would you prove the limit in the previous part?

a. Exponential piece: $C_\infty(y) = 1.0\times10^6 e^{0.1y}$

b. The y^6 decays to zero, so $A_0(y) = 6.0\times10^5 e^{-0.007y}$

c. $\lim_{y\to\infty} \frac{A(y)}{C(y)} = \lim_{y\to\infty} \frac{A_\infty(y)}{C_\infty(y)} = \frac{0.007y^6}{1.0\times10^6 e^{0.1y}} = 0$ because exponentials grow faster than polynomials

d. I would painstakingly use L'Hopital's rule again & again until the ratio was no longer an indeterminate form.

number	value
$0.007\cdot 50^5$	2.18×10^6
$0.007\cdot 50^6$	109.3×10^6
$1.0\times10^6 e^{5.0}$	148.4×10^6
$6.0\times10^5 e^{-0.35}$	0.423×10^6