Biology 5910: Take home final

Do four out of five problems (25 points each). Open books, notes and basic calculators, but no graphics calculators, computer algebra systems, or other human beings (except me). Take only 2.0 hours.

1. If there is a big snowstorm by early December, there's an 80% chance of an inversion. If there isn't a big snowstorm, the chance of an inversion is 40%. Without global climate change (GCC), early snowstorms occurred in 30% of years, but with GCC, this increases to 60%.

   a. Find the probability of an inversion before and after GCC.

   b. Find the probability there was an early snowstorm conditional on there being an inversion before and after GCC.

   c. What additional information do you need to find the probability of climate change conditional on inversion? Make up such information and see what happens.

2. Over the course of 100 years, there are 35 years with early snow and an inversion, 25 with no early snow and an inversion, and 10 years with early snow and no inversion.

    a. Use maximum likelihood to estimate the probability of an inversion given early snow.

    b. Show how you would use maximum likelihood to tell if early snow changes the probability of inversion.

3. Two nearby cities, Pepper Lake City (PLC) and Salt Pond City (SPC) share pollution, but SPC is downwind from PLC. Each can implement emission controls, which reduces asthma cases in that city by 1000. If PLC reduces emissions, SPC will have 500 fewer asthma cases, but if SPC reduces emissions, PLC will have 250 fewer asthma cases.

   a. If the goal is to minimize asthma cases, write this as a game and find the Nash equilibrium.

   b. This neglects economic costs. Suppose the cost is effectively the same as x asthma cases. Find a value of x that produces a prisoner's dilemma or the hawk-dove game, and explain what might happen.

4. During an inversion, the particulate $PM_{2.5}$ level in the air increases by 6.0 μg per day due to emissions, but decreases by 5% each day due to absorbing to surfaces.

    a. Write this as a discrete-time dynamical system.

    b. Find the equilibrium.

    c. Cobweb.

    d. What would happen if the percent reduction got smaller with the higher the particulate level? Sketch a graph that illustrates your conclusion.

5. People are supposed to drive less when the particulate level is high. Let p represent the particulate level, and C the amount of driving people do, where $C = 1$ is maximum driving. Suppose that

$$\frac{dp}{dt} = 5.0C - 0.05p$$

$$\frac{dC}{dt} = 100(1 - C) - p.$$

a. Explain these equations.

b. What would happen to driving if p was fixed at zero? Show this graphically.

c. Find the equilibrium of the system.