

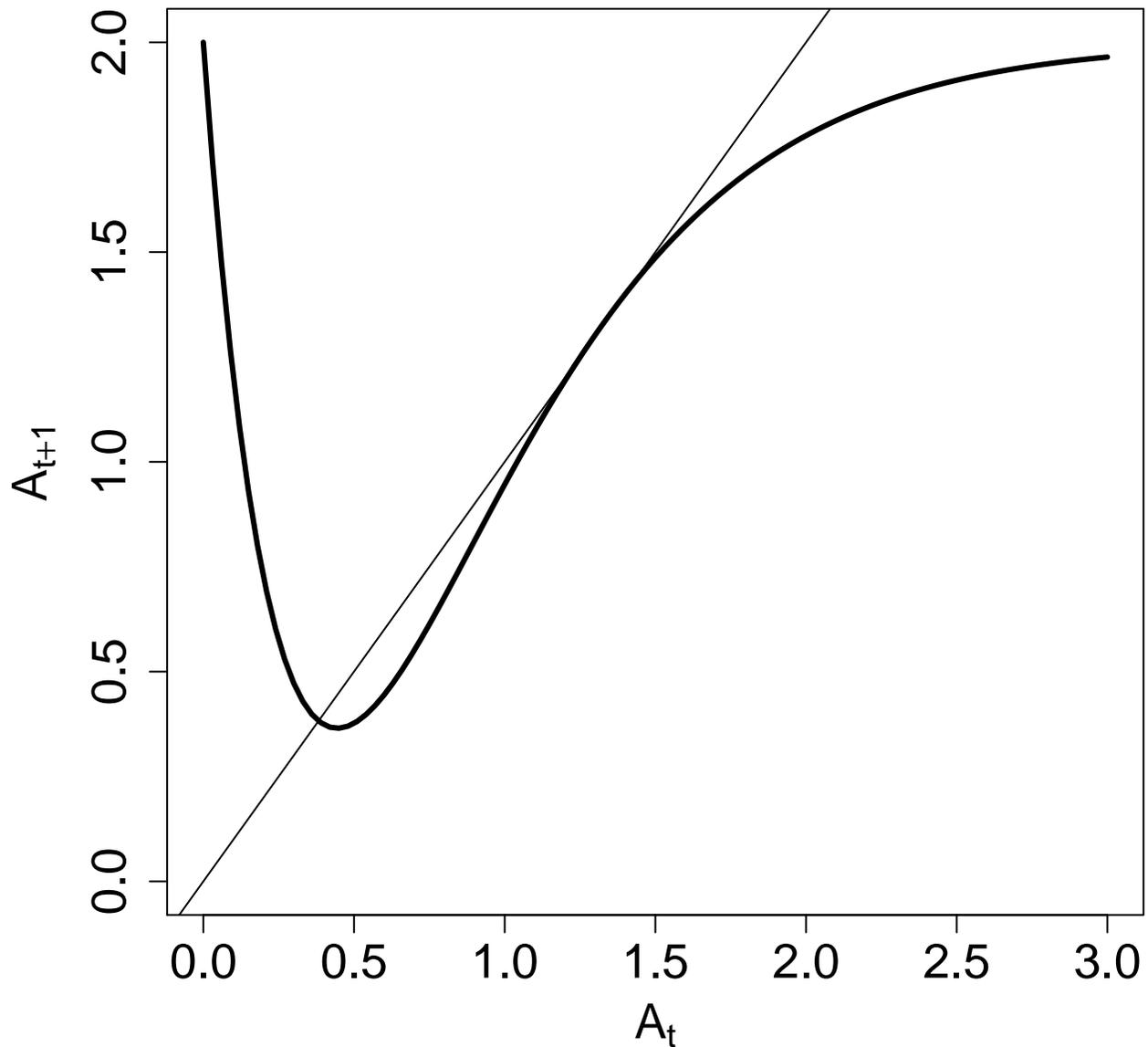
NAME: _____

MATH 5110
Midterm

Do all four problems (25 points each). Write readable answers on the test, but feel free to use or hand in additional paper if necessary. One page of notes, no calculators, no cheating.

1. Loud rumbling noises can be created by the successive breakdown of mechanical components. Suppose a building has two noisy mechanical components, Rumlbers and Grumblers, with R_t and G_t respectively representing the number of broken ones in week t . The vibrations they create lead to further breakdowns.
 - Half (50%) of the broken Rumlbers are fixed each week,
 - Each broken Rumbler generates one new broken Grumbler,
 - Each broken Rumbler generates no new broken Rumlbers,
 - One fourth (25%) of the broken Grumblers are fixed each week,
 - Each broken Grumbler creates b broken Rumlbers,
 - Each broken Grumbler generates no new broken Grumblers.
 - a. Write these rules as a matrix equation.
 - b. What is the largest eigenvalue when $b = 0$?
 - c. For what value of b will the number of broken Rumlbers and remain constant in the long run? What is the limit of the ratio of broken Rumlbers to broken Grumblers?
 - d. For what value of b will the number of broken Rumlbers and Grumblers be equal in the long run? How fast would the noise be increasing?

2. Suppose the aggravation level A_t follows the discrete time dynamical shown in the figure (the thick curve), the graph of $A_{t+1} = g(A_t)$, where $g'(0) = -22.5$. The thin line shows where $A_{t+1} = A_t$.



- Cobweb starting from an aggravation level of 0.2.
- Describe the equilibria of this system.
- Suppose the system were generalized to $A_{t+1} = g(A_t) + c$. What bifurcation is occurring at $c = 0$ (the case shown)?
- What bifurcation will occur with $c > 0$?
- What bifurcation will occur with $c < 0$?
- Draw a bifurcation diagram for this system as we vary the parameter c .

3. Constant low rumbling noises can drive people into a homicidal rage, and seeing others rage can be catching. In particular, suppose the number of people in a homicidal rage in week t is H_t and it obeys the discrete-time dynamical system

$$H_{t+1} = \frac{3}{4}H_t^p + 3 - 2p$$

where p can differ among students.

- a. Suppose first that $p = 1$ for all students. Find the equilibrium number of homicidal ragers and prove that it is stable.
- b. Now suppose that half the students have $p = 1$ and the other half have $p = 2$. Write the discrete-time dynamical system for this case (combine the system from **a** with the system you get by setting $p = 2$).
- c. Find the equilibria and their stability of this new system.
- d. How would you model this if the values of p were distributed uniformly over all values of p between 1 and 2?

4. It is hypothesized that complaining to the authorities can lead to the repair of Rumbler, Grumbler and other noise-producing machinery. Let M_t represent the total number of broken machines in week t and C_t the number of complaints. Suppose that

$$\begin{aligned}M_{t+1} &= \frac{\lambda M_t}{1 + C_t} \\C_{t+1} &= r M_t.\end{aligned}$$

- a. Explain these equations.
- b. Find the equilibria. What are the conditions on the parameters for a positive equilibrium to exist?
- c. Find the stability of any positive equilibria.
- d. What do you plan to do tomorrow?