NAME:_____

MATH 1170: The Final

Do all five problems, points as indicated. Write readable answers on the test, but feel free to use or hand in additional paper if necessary. If you do, **make sure I can find both the answer and how you got it**. You can use four sides of notes, no calculator, and the book. Extra credit indicated with stars.

1. (40 points)

a. Find the period, amplitude, average and phase of the derivative of

$$g(t) = 10\sin(\frac{2\pi(t-1)}{7}).$$

b. Find the period, amplitude, average and phase of the integral of $g(t)$ as defined in part **a**.

c. Find the quadratic approximation to e^x for x near 0 and use it to estimate $\ln(2)$.

d. Find (and explain how you did it)

$$\lim_{x\to\infty} \frac{1+\ln(x+3)+e^{-x}}{10+3\sqrt{x}+\ln(x+3)}.$$

e. *Suppose a function has a positive derivative. What does the function do? (**multiplicative** extra credit: if right, your score is multiplied by 1.01, if wrong, your score is multiplied by zero).

2. (40 points) Little Bob decides to go sledding at Big Hill. His velocity, in meters per second, as a function of time since he let go, is

$$v(t) = 2te^{-0.1t}.$$

a. Find his acceleration as a function of time.

b. When will his velocity be at its maximum?

c. Sketch a graph of his position as a function of time.

d. Will Little Bob ever come to a stop?

e. *Give a numerical value of $\ln(2)$ to at least 3 decimal places.

3. (40 points) The duration of each sled run depends on the run before it. Little Bob and his sister Midge suspect that slow runs are followed by fast runs and that fast runs are followed by slow runs. They discover that T_{t+1}, the time on run $t+1$, is related to T_t, the time on run t, by

$$T_{t+1} = T_t e^{2.0-0.02T_t^2}$$

where T is measured in seconds.

a. Find the equilibrium or equilibria.

b. Find their stability.

c. Sketch a cobweb diagram, starting from a very slow run.

d. Were Little Bob and Midge right about the pattern of slow and fast runs? Explain.

e. *The name "Euler" rhymes with a) cooler, b) boiler, c) Buehler, d) feeler, e) all of the above.

4. (40 points) Little Bob and Midge decide that they should maximize their rate of having fun. Each sled run gives one unit of fun (one fundegree) and takes 10 seconds. However, climbing up the hill between runs subtracts from this fun according to

$$\text{reduced fun if time spend climbing is } t \text{ seconds} = f(t) = \frac{(t-5)^2}{25}$$

where t is measured in seconds.

a. For a given climbing time t, how much fun do the kids have on a given sled run (decrementing the sledding fun by the climbing misery)? What set of climbing times give net positive fun?

b. What amount of climbing time maximizes the fun per trip? Explain this result in words.

c. Find the rate of having fun (dividing total fun by total time for each trip).

d. How would you find the climbing time that maximizes the rate? Is it more or less than the time that maximizes the fun per trip?

e. How does this differ from a rate maximization problem involving bees and flowers? Would it be possible to use the marginal value theorem to solve part **d**?

f. *Solve $\ln(x) = 2$. How is this different from finding $\ln(2)$?

5. (40 points) The happy shrieks of the children unfortunately trigger an avalanche, and they are buried under 2.0 meters of snow. The density of the snow x meters above the ground is

$$\rho(x) = x^3 - 4x^2 + 3x + 2,$$

measured in thousands of kilograms per meter. Little Bob's cross-sectional area is 0.25 square meters.

a. What is the total mass of the snow pressing down on Little Bob?

b. What is the average density of the snow? Does this value make sense (in comparison with the density of water which is 1 gram per cubic centimeter)?

c. Use the left hand estimate with $\Delta x = 1$ to estimate the total mass of the snow. Why does it differ as it does from the exact answer?

d. *What is calculus? Your answer will be graded for content, brevity, grammar and spelling. Answers need not exceed 7 words in length and should not exceed 77 words.